MATH MATTERS

Department of Mathematics Cornell University Ithaca NY December 2013

Letter From The Chair, Laurent Saloff-Coste

Over the summer, the College of Arts and Sciences welcomed its new Dean, Gretchen Ritter. Also, joining the Dean's College leadership team this year is Scott MacDonald, Professor of Philosophy, who is the Senior Associate Dean overseeing the Department of Mathematics.

The next ten years promise to be an exciting and challenging time marked by several unique initiatives which include the development of the New York Tech Campus and a college-wide focus on faculty renewal to address the issue of replacing a large number of senior faculty members transitioning towards retirement. Over the last three years, Mathematics has welcomed seven new faculty and this fast paced hiring effort will need to continue for the foreseeable future. How well we succeed will impact the future of the department, the quality of many other programs across the University, and Cornell's ability to fulfill its core undergraduate education mission.

Last June saw the retirement of two faculty who spent most of their career at Cornell, Leonard Gross and Alfred Schatz. Both are now Professor Emeriti. Sadly in July, we lost a dear friend and colleague, Lars Wahlbin, who was our Director of Undergraduate Studies. For many years, Lars and Al formed a very successful research team. Our efficient Chair's assistant, Brenda Smith, left for Massachusetts this past summer and was replaced by Kelly Patwell who, in the short time she has been with us, has already made herself indispensable.

Welcome to Senior Lecturer Quincy Loney who replaces Patty Alessi at the Learning Strategy Center, as well as to our new Assistant Professor, David Zywina, and to Postdoctoral Associates Amy Cochran, Raul Gomez, John Pike and Farbod Shokrieh. Their fresh perspectives and energy are essential to the success of the department.

The gifts of individuals who have contributed to our endowment funds and our general gift fund allow us to sustain and enhance many key activities including working with local schools to enhance K-12 mathematics education, supporting research experiences for undergraduates, support for our graduate program and activities, plus bringing research visitors to the department. Thank you all for you generosity.

I look forward to reporting on our progress in next year's sesquicentennial edition of Math Matters!

John Guckenheimer wins 2013 Steele Prize

By Susan Kelley of The Cornell Chronicle

For his contributions to the field of dynamical systems, John Guckenheimer, the A.R. Bullis Professor of Mathematics, was awarded the 2013 AMS Leroy P. Steele Prize for Mathematical Exposition. Presented annually by the American Mathematical Society, the Steele Prize is one of the highest distinctions in mathematics.

The prize was awarded Jan. 10 at the society's meetings in San Diego. Also honored with the prize was Guckenheimer's co-author, Philip Holmes of Princeton University.

Guckenheimer and Holmes were honored for their book, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector*

Continued on page 2.

John Guckenheimer wins 2013 Steele Prize

By Susan Kelley of The Cornell Chronicle

Continued from page 1.

Fields (Springer-Verlag, 1983), the first treatise on the modern theory of dynamical systems. It explores the application of methods from dynamical systems and bifurcation theories to the study of nonlinear oscillations.

The understanding of dynamical systems underwent a rebirth in the 1960s and 1970s thanks to mathematicians and other theoreticians, and applied scientists such as engineers and experimental physicists, the prize citation says. "Not surprisingly, it was difficult for the two communities to know about each other's work until the publication of the now-classic text by Guckenheimer and Holmes. Thirty years later this book remains in wide use as a standard text for graduate-level courses in mathematics departments and throughout the sciences and engineering, and Chinese and Russian translations have appeared," the citation said.

The authors met in 1976 when rapid advances in dynamical systems theory were stimulating experimental work that demonstrated the usefulness of the theory in explaining empirical phenomena across the sciences and engineering, Guckenheimer said. "We saw a real need for a book that made the new mathematics accessible to a broad audience," he said. "We tried hard to explain mathematical concepts and arguments in their simplest manifestations while relying on as little formal training as seemed feasible. It helped that we came to the interface between mathematics and the physical sciences from opposite sides."

During the past 15 years, Guckenheimer's research has investigated dynamical systems with multiple time scales and associated numerical methods. He has also continued to investigate the use of dynamical systems theory in diverse areas, notably in neuroscience and animal locomotion. He was a 1984 Guggenheim fellow and is a fellow of the American Academy of Arts and Sciences, the American Association for Advancement of Science, the American Mathematical Society, and the Society for Industrial and Applied Mathematics, where he served as president 1997-98.

Updates from the Math Library

By Jill Wilson

The past year has been vital and busy for the Mathematics Library at Cornell.

A committee was appointed to assess the collections and services in the Mathematics Library (Chaired by Richard Shore and Steve Rockey with Keith Dennis, Tara Holm, Kathryn Lindsey, Steven Strogatz, Marten Wegkamp and Jill Wilson). Data was collected on the usage of materials and services offered by the library, including study space, literature access and instruction. The conclusions pointed to the fact that the Mathematics Library is a necessary physical space for study and literature access here on campus. The mathematics print collection circulates at a very high rate with large percentage of the collection in use and remains one of the highest circulating collections here at Cornell.

In addition to collections, the study space in the Math Library is seen as valuable and pertinent to all of campus. Many students who visit the library are from other related disciplines, not just mathematics. The centralized location is convenient to those taking classes in Malott or adjacent buildings and serves as a refuge for those seeking quieter study spaces than what other spaces offer, such as the more social Mann Library spaces or the Clark Hall atrium. The closure of the Physical Sciences Library also led to a rise in traffic to the Math Library for study space and high-circulating items in the physical sciences that were moved to this library.

Along with analyzing data, the committee made some recommendations. The first is the creation of a

Continued on page 4.

A Vexing Math Problem Finds an Elegant Solution

By Anne Ju of The Cornell Chronicle

A famous math problem that has vexed mathematicians for decades has met an elegant solution by Cornell researchers. Graduate student Yash Lodha, working with Justin Moore, professor of mathematics, has described a geometric solution for the von Neumann-Day problem, first described by mathematician John von Neumann in 1929.

Yash Lodha

Lodha presented his solution at the London Mathematical Society's Geometric and Cohomological Group Theory symposium in August, and has submitted the work to a journal. "People were very excited by this," Lodha said. "[The solution] is natural and compelling enough to study for its own sake."

Lodha works in the field of geometric group theory. A group is a mathematical construct that describes the notion of the symmetries of an object, whether it's a physical object or a theoretical space. For example, a polygon has rotational as well as reflectional symmetries, all of which, together with the operation of composition, form what's called a finite group, because the polygon can be described as a finite sequence of operations that reflect its symmetries.

Formally, a group can be described as words in an alphabet together with a set of rules that are called "relations." Group theorists, Lodha said, are like biologists who classify species; mathematicians try to categorize groups that have properties A, B or C – but is there one that has A but not C?

The inspiration for Lodha's work originated in the early 20th century, when mathematicians first proved that a ball that exists in three-dimensional space can be chopped into a finite number of pieces – "like tearing up a piece of paper without stretching or squeezing," Lodha explained – and can be reassembled, like a jigsaw puzzle, into two balls, each the size of the original ball. This is known as the Banach-Tarski paradox.

von Neumann, in studying this paradox, was the first to describe the reason behind it: He attributed it not to the geometry of 3-D space, but to the algebraic properties of the symmetries inherent to the sphere. He was the first to isolate this property, which mathematicians today call "non-amenability."

von Neumann further observed that if a group contains free groups, which are groups that have a finite alphabet and no rules, then it must be non-amenable. He posed the question of whether the opposite is true – are there groups that do not contain free groups and are also non-amenable? The problem, later popularized by M.M. Day, waited another 40 years before mathematician Alexander Olshanskii cracked it, although Olshanskii's group had an infinite set of rules.

Another two decades went by before Olshanskii and Mark Sapir supplied another solution in response to the von Neumann-Day problem. This time, their example was governed by a finite, but astronomically large set of rules – close to 10^{200}. It also lacked a natural geometric model. So mathematicians probed further for a group with a finite set of rules, that is non-amenable and does not contain free groups.

For the first time, Lodha describes a group that has only nine rules, a natural geometric model, is non-amenable and does not contain free groups.

Advances in mathematics are almost always incremental and build upon previous work, Lodha said. To complete this work, among his most valuable insights was one first described by the late Bill Thurston, Fields medalist and Cornell's Jacob Gould Schurman Professor of Mathematics, which involved a way of expressing the group in a different light, as a "continued fractions model."

Lodha's work also builds heavily on work by Nicolas Monod, who constructed a geometrically oriented, but not finitely presented, counterexample to the von Neumann-Day problem. Lodha and Moore's contribution was to isolate a finitely presented subgroup, with only nine relations, of Monod's example.

Further work on the group, which doesn't yet have a name, could make the solution to the von Neumann-Day problem even stronger: by isolating stronger finiteness conditions for proving that the group has a finite number of rules.

Justin Moore

The research was supported by the National Science Foundation.

Updates from the Math Library

By Jill Wilson

Continued from page 2.

graduate student reading room in the library itself. This room, inspired by the new graduate study room in Olin library, would be accessible for graduate students only since they currently don't have a specific place to retreat to in Malott Hall. Another recommendation is the creation of a learning commons, or learning center, in another, separate part of the library to serve the "up and coming" research trends occurring in social media and elsewhere. This space would include multimedia possibilities such as television/PC monitors to connect users to the outside world or research through social media, interactive programming and other educational opportunities for the Cornell math community.

The library continued to increase outreach efforts to the math community through various events. In May, the library held a book talk featuring Daina Taimina, adjunct associate professor in mathematics here at Cornell. Her book *Crocheting Adventures with Hyperbolic Planes* won the 2012 Euler Book Prize awarded by the Mathematical Association of America, and features creative ways to demonstrate the complexities of these geometric models. Daina brought with her several of her crocheted creations to show book talk attendees; as well as speak about how her craft helped her understand hyperbolic planes.

The library kept Daina's crocheted art on display for reunion, where alumni got to see and touch her work firsthand. Two of her larger pieces were actually hanging from the library ceiling right at the entrance! Along with the art display were math books that were about art and how the two disciplines intersect.

In August, the librarians welcomed new students via several events. All were invited to the EMPSL open house (EMPSL stands for Engineering, Math and Physical Sciences Libraries). This took place in Carpenter Hall during the first week of classes. Jill Wilson (Outreach Coordinator for EMPSL) led a session for new graduate students on orienting them to the library. The instruction session included an introduction to the math library website, the collections in the library and a quick demo of MathSciNet.

Earlier this October, the math club partnered with the library to show the film *Good Will Hunting* to club members. The Math Library owns many films, ranging from documentaries and interviews to popular Hollywood produced films. The math club and the library plan on doing another event in the spring, to be determined.

Also in October, Jill Powell, Engineering Librarian, and Jill Wilson gave a presentation on predatory publishers during Open Access Week. The presentation included ways to spot predatory journals and what to do when you are solicited by sub-par open access publications.

We look forward to another busy and exciting year in 2014.

www.mathematics.library.cornell.edu

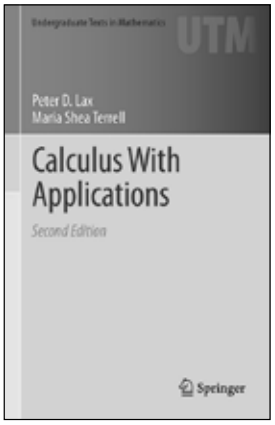
Saturday Workshops for Math Teachers

By Mary Ann Huntley

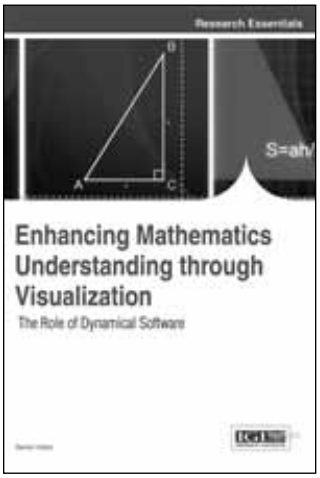
Since 1985, during each academic year the Cornell Mathematics Department has offered a series of four full-day workshops for secondary mathematics teachers. During the 2012-2013 academic year, several faculty members (Tara Holm, Lionel Levine, Ravi Ramakrishna, Laurent Saloff-Coste), visitors (Nate Eldredge, Ana Rita Pires, Tasia Raymer, Benjamin Steinhurst, Rodrigo Trevino), and graduate students (Mark Cerenzia, Mathov Murugan, Tianyi Zheng) used MATH 5080 as a forum for sharing their mathematical interests and curiosities. In the photo to the right, Tara Holm is demonstrating for teachers how to turn her trousers inside out without taking them off!

Faculty Contributions to New Books

By Mary Ann Huntley



Peter Lax (New York University) and Maria Shea Terrell (Senior Lecturer of Mathematics and Director of Teaching Assistant Programs) consider approximation and calculation to be central issues in using mathematics, and that calculus gives the basic ideas and language for both. They have incorporated these ideas in their new book, *Calculus With Applications*, which is available from Springer. This is a VERY different point of view from most garden-variety calculus books that focus their efforts on asking students to work as though they evaluate expressions exactly—the whole meaning of the deltas and epsilons is never framed in terms of tolerance for error (epsilon) and precision of the input (delta). Draft versions of the book have been used for the past three years for Honors Calculus II (MATH 1220). Terrell and Lax are currently writing a follow-up book on multi-variable calculus.



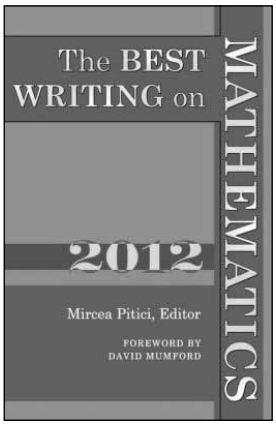
Helping students understand mathematical concepts through visualization, particularly through the use of dynamical software, is common practice in many mathematics classrooms. Three Cornell Mathematics Faculty share their work in this arena by contributing chapters to a new book, *Enhancing Mathematics Understanding through Visualization: The Role of Dynamical Software* (IGI Global, 2013).

- *Technology and Differential Equations*, by John Hubbard (Professor)
- *Nonlinear is Essential, Linearization is Not Enough, Visualization is Absolutely Necessary*, by Beverly West (Senior Lecturer, retired)
- *Applets for Mathematical Learning*, by Robert Terrell (Senior Lecturer)

Readers of this book will find many ideas, and much evidence, for how dynamic software can make mathematical concepts much easier for students to assimilate. This book can be read online at: **www.math.cornell.edu/VisualizationBook**

A New Book Gives Math Everyday Meaning

By Anne Ju of The Cornell Chronicle



The 2012 volume *The Best Writing on Mathematics*, edited by Cornell's Mircea Pitici, helps the average person understand how math relates to daily life.

For the third straight year, Pitici, a graduate student in mathematics, has edited the volume, published by Princeton University Press, that collectively answers the question posed to many mathematicians: "What is it you do?"

The collection of essays touches on the history and philosophy of the field, math education and presentations of mathematical ideas. Among them: the intricacies of the distribution of prime numbers, and a lay-friendly explanation of octonions, which are a strange type of algebra in which "numbers" are 8-tuples of an ordinary number, according to the book's foreword by mathematician David Mumford.

Other pieces divulge how math can be used in science and in life: in dancing, as a traveling salesman, in search of marriage and full-surround photography.

Pitici has taught math and writing seminars at Cornell, Ithaca College and Wells College. He received a teaching award from the Cornell Department of Mathematics in 2011, as well as the Buttrick-Crippen Scholarship awarded by the Knight Institute of Writing in the Disciplines in 2008.

He holds a bachelor's degree in mathematics from the University of Bucharest, Romania, and a master's degree from Cornell, and is working toward a doctorate in mathematics education.

Research Experience for Undergraduates

by Robert Strichartz (director)

The Summer of 2013 marked the 20th year for the math department's REU program, supported by the National Science Foundation. This summer saw 21 students work in three project areas: 1) Analysis on Fractals, directed by Robert Strichartz with assistance from Cornell graduate student Baris Ugurcan, an alum from an earlier REU program; 2) Generating Sets for Finite Groups, directed by R. Keith Dennis with assistance from Chicago graduate student Paul Apisa; and 3) High Dimensional Data Analysis, directed by Matthew Hirn, an assistant professor from Yale, as well as a former Cornell math major.

The students in the Analysis on Fractals group worked on 5 separate projects. Robert Ravier, a graduating Cornell math major who is now a graduate student at Duke University, completed a paper on sampling theory with average values on the Sierpinski gasket that originated in his senior thesis. Ilse Haim (University of Maryland) and Travis Westura (Cornell University) studied sandpile models on graph approximations to the Sierpinski gasket, and will be giving a talk on their work at the annual AMS meeting in Baltimore in January. Sandpile models are a popular topic at Cornell, appearing in the research of three faculty members, Lionel Levine, Farbod Shokrieh, and Charles Smart (arriving in Fall 2014).

Nick Ryder (Rice University) and Ben Li (Chinese University of Hong Kong) together with Baris Ugurcan completed a paper on constructing functions on the Sierpinski gasket given a finite set of values so as to minimize certain natural norms. One of the consequences of their construction is a discrete analog of a continuous theorem discovered by an REU student (Justin Owen) several years ago. Nick will also be talking about this paper at the annual AMS meeting.

Denali Molitor (Colorado College) and Nadia Ott (San Diego State University) studied the use of Peano curves to constrict Laplacians of certain fractals. Classical Peano (space filling) curves are usually regarded as mindboggling curiosities with no serious applications. In this work the analogs of Peano curves play a vital role, and they provide a new way of visualizing the eigenfunctions of the Laplacian, even in nonfractal settings such as the equilateral triangle. The inspiration for this approach comes from the work of several REU students from previous years who studied Lapacians on Julia sets. In addition to working to complete a paper, the students have produced a website, **www.math.cornell.edu/~dmolitor**, that has impressive graphics.

Mengyuan Zhang (Jacobs University) and Gamal Mograby (University of Berlin) studied Anderson localization in quantum mechanics for fractal Laplacians. By forming a Schrodinger operator consisting of a standard Laplacian plus a random potential, Anderson noticed that there are many highly localized eigenstates. This observation and its consequences won him a Nobel Prize in Physics, but a full mathematical understanding is still incomplete. Mengyuan and Gamal based their work on a recent paper by a physicist (Filoche) and a mathematician (Mayboroda) that explains how to find the regions of localization in the classical setting, and extended it to fractal Laplacians. Some spectacular images are visible on their website: **www.math.cornell.edu/~myzhang**.

Six students participated in the "Generating sets for finite groups" project, including 2 from Cornell, Benjamin Fayyazuddin-Ljungberg and Eric Primozic. Paul Apisa (graduate student at the University of Chicago), a participant in the corresponding REU at Cornell in summer 2011, served as assistant. The basic problem studied was determining how generating sets of finite groups behave. As a motivation one has the example for finite dimensional vector spaces over a finite field. Even such things as the dimension of a vector space turn out to have at least a half-dozen or more reasonable and different generalizations to finite groups. It is thus interesting to see how these are related to each other as well as how they behave when one asks questions that have easy and simple answers for vector spaces. Computers were used to perform experiments of some common groups such as PSL(2,p), symmetric, alternating, and other linear groups over finite fields to determine how the various invariants behaved. The students made significant progress in a number of cases that will result in the publication of 2 or 3 papers.

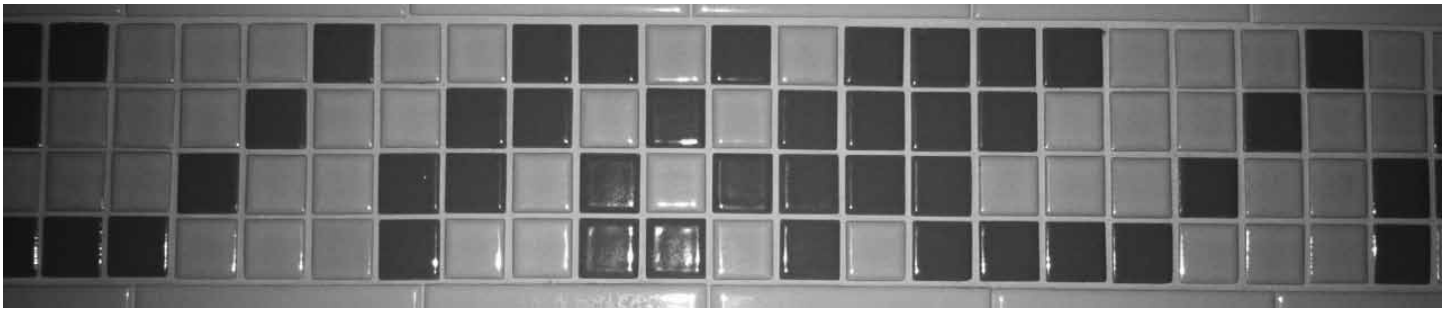
"High Dimensional Data Analysis" had six students working across five projects. Wendy Zeng (Cornell

Continued on page 7.

Puzzle

BY DEXTER KOZEN

This photo shows a repeated pattern of bathroom tiles in my shower at home. Can you figure out what mathematical structure it represents? The answer can be found at **www.math.cornell.edu/~2013puzzle**



Research Experience for Undergraduates

BY ROBERT STRICHARTZ (DIRECTOR)

Continued from page 6.

University) worked on historical economic data, which consisted of a large number of economic indicators (e.g., GDP) across numerous country/year pairs (e.g., USA/2003). She was interested in which countries have similar economic patterns and which economic indicators are correlated. Wendy employed diffusion based tree structures to organize both the country/years and the indicators, leveraging one organization against the other in a back and forth scheme that yielded some very cool low dimensional geometric patterns.

Christian Smith (Macalester College) worked on a project involving dynamical systems, and in particular the Lotka-Volterra equations used in biology to model the interaction of different species. Christian obtained a low dimensional embedding of the dynamical system that organized trajectories according to their behavior. From there he studied how the low dimensional embeddings changed as he adjusted the parameters of the system (the parameters govern how the species interact).

Nicholas Marshall (Clarkson University) studied the heat equation with time dependent Laplacian, which governs how heat spreads over a time evolving manifold. Just as random walks can be used to approximate heat flow on a static manifold, Nick was interested in coming up with an analogous approximation for the time-dependent case. The solution was to use non-time-homogeneous Markov chains, which can be thought of as random walks whose transition probabilities change with each step.

Keyi Wu (Cornell University) studied the use of minimum path distances to organize flow cytometry data from cell biology. Through a process called gating, flow cytometry measurements are used to organize various cell populations for the diagnosis of medical disorders. Keyi has efficiently automated this gating process by clustering the data according to a specific type of minimum path distance that can isolate strangely shaped regions of high density, which usually correspond to specific populations.

Finally, Ariel Herbert-Voss (University of Utah) and Frederick McCollum (University of Arkansas) worked together on a project involving interpolation of data by smooth functions. Given function values and partial derivatives, the goal was to compute the interpolant whose derivative has minimal Lipschitz constant. It turns out that one can construct such an interpolant (but not easily), and to- gether Ariel and Derick translated this construction into an efficient algorithm by utilizing a litany of tools, including algorithms for computing convex hulls, power diagrams, triangulations, and tree structures.

Next year the REU program will continue with projects on Analysis on Fractals (Bob Strichartz), Nonlinear heat equations (Xiaodong Cao) and Graphs, chip-firing games, and algebraic geometry (Farbod Shokrieh).

Cornell University
Department of Mathematics

310 Malott Hall
Ithaca, NY 14853-4201

Mathematics Department Endowments & Gifts

We are grateful to alumni, friends, and family for their generosity in supporting our endowments or providing other gifts and donations to the department.

The **Ruth I. Michler Memorial Prize,** established by Gerhard and Waltraud Michler of Essen, Germany, in memory of their daughter, provides funding for the Ruth I. Michler Memorial Prize of the Association for Women in Mathematics. The awardee spends a semester here without teaching obligations.

The **Chelluri Lecture Series** was established by Raju Chelluri's parents in his memory. Funds are used to invite distinguished mathematicians to give annual lectures.

The **Michael D. Morley Senior Prize in Mathematics** is presented annually to an Ithaca High School student who has excelled in mathematics and who has demonstrated originality and innovative power in mathematics.

Teaching Awards for Graduate Students and faculty were created in 2001. Prizes are awarded to graduate students.

The **Colloquium Endowment Fund** was instituted to invite distinguished scientists to speak at the Oliver Club seminars. (See www.math.cornell.edu/~oliver/.)

The **Eleanor Norton York Endowment** was established in honor of Eleanor Norton York to recognize outstanding graduate students in both Astronomy and Mathematics.

The **Faculty Book Endowment** is dedicated to providing the Cornell community with access to one of the world's finest collections of mathematics books and publications.

The **Israel Berstein Memorial Fund** was established in honor of Israel Berstein, a professor in this department from 1962-1991. The memorial fund is intended to help young mathematicians in the field of topology.

The **Logic Endowment** was started with a generous gift from a former Cornell undergraduate to support promising logic students.

The **Robert John Battig Endowment** was established by his parents after his untimely death. Robert was awarded a January 1998 Ph.D. in mathematics. The fund provides an annual prize to an outstanding continuing graduate student in mathematics at Cornell.

If you would like to contribute, please make your check payable to Cornell University, indicate the endowment, or that it is a gift in support of Mathematics, and send it to:

Department of Mathematics Endowments & Gifts
310 Malott Hall, Cornell University
Ithaca, NY 14853-4201

Gifts can also be made online at
www.giving.cornell.edu