

CORNELL UNIVERSITY MATHEMATICS DEPARTMENT SENIOR THESIS

**REPRESENTATION THEORETIC EXISTENCE PROOF FOR
FISCHER GROUP Fi_{23}**

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May 2008

A THESIS PRESENTED IN PARTIAL FULFILLMENT
OF CRITERIA FOR HONORS IN MATHEMATICS

BACHELOR OF ARTS, CORNELL UNIVERSITY

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ABSTRACT

In the first section of this senior thesis the author provides some new efficient algorithms for calculating with finite permutation groups. They cannot be found in the computer algebra system MAGMA, but they can be implemented there. For any finite group G with a given set of generators, the algorithms calculate generators of a fixed subgroup of G as short words in terms of original generators. Another new algorithm provides such a short word for a given element of G . These algorithms are very useful for documentation and performing demanding experiments in computational group theory.

In the later sections, the author gives a self-contained existence proof for Fischer's sporadic simple group Fi_{23} of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ using G. Michler's Algorithm [11] constructing finite simple groups from irreducible subgroups of $\text{GL}_n(2)$. This sporadic group was originally discovered by B. Fischer in [6] by investigating 3-transposition groups, see also [5]. This thesis gives a representation theoretic and algorithmic existence proof for his group. The author constructs the three non-isomorphic extensions E_i by the two 11-dimensional non-isomorphic simple modules of the Mathieu group \mathcal{M}_{23} over $F = \text{GF}(2)$. In two cases Michler's Algorithm fails. In the third case the author constructs the centralizer $H = C_G(z)$ of a 2-central involution z of E_i in any target simple group G . Then the author proves that all conditions of Michler's Algorithm are satisfied. This allows the author to construct G inside $\text{GL}_{782}(17)$. Its four generating matrices are too large to be printed in this thesis, but they can be downloaded from the author's website [10]. Furthermore, its character table and representatives for conjugacy classes are computed. It follows that G and Fi_{23} have the same character table.

ACKNOWLEDGEMENTS

The author would like to thank professor Gerhard O. Michler for all his mathematical support and encouragement, and also professor R. Keith Dennis for his guidance in the Fall term of 2007.

The author also acknowledges financial support by the Hunter R. Rawlings III Cornell Presidential Research Scholars Program for participation in this research project.

This work has also been supported by the grant NSF/SCREMS DMS-0532/06.

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0. INTRODUCTION

A simple group G is called sporadic if it is not isomorphic to any alternating group A_n or any finite group of Lie type, see R. W. Carter [3]. Until recently there was no uniform construction method for the known twenty six simple sporadic groups. In [11] a uniform construction method is given for constructing finite simple groups from irreducible subgroups of $\text{GL}_n(2)$. For technical reasons it cannot construct the two largest known sporadic simple groups because there is no computer which can hold all their elements. But the other twenty four known sporadic simple groups can be constructed by Michler's Algorithm. In this thesis it is applied to provide a new self-contained existence proof for Fischer's sporadic simple group Fi_{23} .

In 1971 B. Fischer [5] found three sporadic groups by characterizing all finite groups G that can be generated by a conjugacy class $D = z^G$ of 3-transpositions, which means that the product of two elements of D has order 1, 2, or 3. He proved that besides the symmetric groups S_n , the symplectic groups $\text{Sp}_n(2)$, the projective unitary groups $\text{U}_n(2)$ over the field with 4 elements and certain orthogonal groups, his two sporadic simple groups Fi_{22} and Fi_{23} , and the automorphism group Fi_{24} of the simple group Fi'_{24} describe all 3-transposition groups, see [6]. For each 3-transposition group $G = \langle D \rangle$ Fischer constructs a graph \mathcal{G} and an action on it. As its vertices he takes the 3-transpositions x of D . Two distinct elements $x, y \in D$ are called to be *connected* and joined by an edge (x, y) in \mathcal{G} if they commute in G . He showed that each of the groups considered in his theorem has a natural representation as an automorphism group of its graph \mathcal{G} . Unfortunately, Fischer's proofs are only published in his set of lecture notes of the University of Warwick [6]. See also [1], for a coherent account on Fischer's theorem.

In [6] Fischer gave the first existence proof for Fi_{22} , Fi_{23} , and Fi_{24} , by constructing the three graphs on which the groups act as automorphisms. Eighteen years later, M. Aschbacher proves in [1] the existence of Fi_{24} and hence also Fi_{22} and Fi_{23} , using a quotient of the normalizer of a cyclic subgroup of order 3 in the Monster simple group M , see [1], p. 5. However, both approaches do not allow specific calculations with elements in these groups nor do their methods generalize to arbitrary finite simple groups.

The purpose of this thesis is to provide a new existence proof for Fi_{23} , which has two advantages over the previous proofs. Fischer's proof doesn't generalize to all simple groups, because not all simple groups can be described by 3-transposition groups. Aschbacher's proof obviously doesn't generalize nor does it provide access to explicit computation with elements in the resulting groups, because the Monster group M doesn't have a permutation representation or a matrix representation of small enough degree; the smallest known faithful permutation representation of M wouldn't fit into any modern super-computer, and the smallest degree of matrix representation of M is about 183,000. Dealing with dense 183,000 by 183,000 matrices is practically impossible for currently existing computers.

The new proof uses representation theoretic and algorithmic methods, mainly based on the Algorithm 2.5 of [11], which is also stated in section 2. The second part of Algorithm 2.5 of [11] is not repeated in this thesis, because it is identical to Algorithm 7.4.8 of [12]. Using this algorithm, the author constructs Fi_{23} as a subgroup of $\text{GL}_{782}(17)$. From the 782-dimensional matrix representation, the author also constructs a faithful permutation representation of Fi_{23} of degree 31671, by which we can actually compute with elements and subgroups and check the performed

calculations using the high performance computer algebra system MAGMA. The character table and representatives for conjugacy classes of Fi_{23} are also computed by means of this permutation representation.

The other strong point of this algorithmic proof is that this method generalizes to all simple groups (not having Sylow 2-subgroups which are cyclic, dihedral or semi-dihedral), see [12]. In [9] the author and G. Michler construct Fi_{22} and Conway's sporadic group Co_2 simultaneously. The author and G. Michler are also working on the simultaneous constructions of Co_1 , Janko's sporadic group J_4 , and Fi'_{24} . In [12] and recent work to which the author's joint article [9] with Michler and also the further work belongs, G. Michler and coauthors construct all known sporadic groups except the Baby Monster and the Monster. These methods allow a systematic search for simple groups.

Here is the summary of each section. Section 1 contains the description and MAGMA implementation of the author's short-word algorithms. In section 2, the author states Michler's Algorithm 2.5 of [11], which is a main tool of the construction of Fi_{23} in this thesis.

Section 3 shows the author's construction of the extensions of the Mathieu group \mathcal{M}_{23} by its two non-isomorphic irreducible representations V_1 and V_2 of dimension 11 over $\text{GF}(2)$. In this thesis it is shown that there is a uniquely determined non-split extension E of \mathcal{M}_{23} by V_1 . Michler's Algorithm 2.5 in [11] is applied to E for the construction of Fi_{23} in the next sections. However, its application to the split extensions E_1 and E_2 of \mathcal{M}_{23} by V_1 and V_2 does not lead to any result.

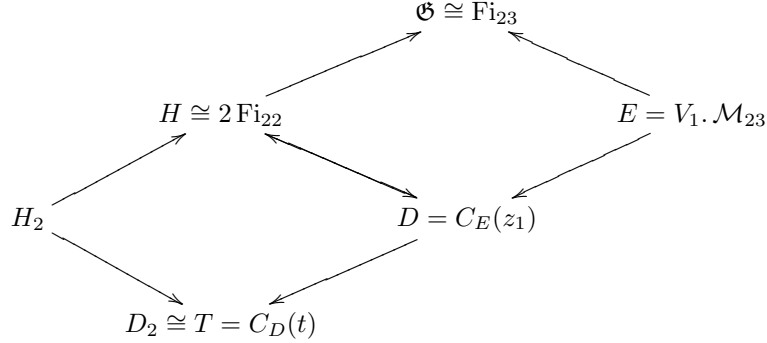
Section 4 contains the author's construction of the centralizer H (unique up to isomorphism) of a 2-central involution z_1 of E in any target simple group G . In particular, it is shown that $Z(H) = \langle z_1 \rangle$ and $H/Z(H) \cong \text{Fi}_{22}$. Taking a 2-central involution z_1 in E and calculating $D = C_E(z_1)$ the author finds a suitable normal subgroup Q in D which enables him to construct a group H with center $Z(H) = \langle z_1 \rangle$ of order 2 such that $H/Z(H) \cong \text{Fi}_{22}$. It is shown that $E, D = C_E(z_1), H$ satisfy all conditions of Algorithm 2.5 of [11], stated in Algorithm 2.1 in section 2.

In order to construct $H \cong 2\text{Fi}_{22}$, the author quotes a result on Fischer's sporadic simple group Fi_{22} from his joint article [9] with G. Michler. This was necessary because the implementation of Holt's Algorithm into MAGMA was not able to construct a central extension of Fi_{22} by a cyclic group of order 2. Thus, the author constructs another amalgam $H_2 \leftarrow D_2 \rightarrow D$, where $D_2 = C_D(t)$ for some involution t and $H_2 = 2H(\text{Fi}_{22})$, see Propositions 4.1 and 4.2 and Theorem 4.3. The free product $H_2 *_{D_2} D$ with amalgamated subgroup D_2 has a 352-dimensional faithful irreducible representation over $\text{GF}(17)$, whose corresponding matrix group \mathfrak{H} is proved to be isomorphic to the 2-fold cover 2Fi_{22} of Fi_{22} , see Theorem 4.5.

In section 5 the author finally constructs the simple target group \mathfrak{G} as a matrix group inside $\text{GL}_{782}(17)$, by applying Algorithm 7.4.8 of [12] to the amalgam $H \leftarrow D \rightarrow E$. In particular, it has been shown that \mathfrak{G} has a same character table as Fi_{23} as stated in [4]. The amalgam $E \leftarrow D \rightarrow H$ has a unique compatible pair of degree 782 over $\text{GF}(17)$ which is not multiplicity-free at the D -level, see Theorem 5.1. So the author uses Thompson's Theorem 7.2.2 of [12] in the application of Step 5(c) of Algorithm 7.4.8 of [12]. It is shown that the free product $H *_D E$ with amalgamated subgroup D has exactly one (unique up to isomorphism) irreducible representation of degree 782 over $\text{GF}(17)$ satisfying the Sylow 2-subgroup test, see Theorem 5.1. The corresponding matrix group \mathfrak{G} in $\text{GL}_{782}(17)$ is proved to be a simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ which has a 2-central involution

3 such that $C_{\mathfrak{G}}(3) \cong H$, see Theorem 5.1. Furthermore, the author constructs a faithful permutation representation of \mathfrak{G} , and then the character table of \mathfrak{G} . It agrees with the one of Fi_{23} as stated in the Atlas [4].

The following diagram summarizes the author's construction of Fi_{23} given in sections 4 and 5.



All the performed demanding calculations for the given existence proof were only possible because of the implementations of the author's new algorithms described in the first section. They are very general and can be used in the course of mathematical research in computational group theory. The author's code works well in the computer algebra system MAGMA. For documentation of the performed calculations it is often crucial to get short-word generators for a certain subgroup of a group G with a given set of generators: $G = \langle g_1, g_2, \dots, g_n \rangle$. Let S be a subgroup G . We want to get a generating set for S as short words in terms of the original generators g_1, g_2, \dots, g_n .

For example, $S = \langle g_1 g_3 g_6, g_2, g_4 g_5 \rangle$. We want relatively small number of generators, and the lengths of words to be short.

MAGMA's inverse word map function (often) provides a generating set in terms of given generators, but unfortunately consisting of terribly lengthy words. Reiner Staszewski, who was a former research assistant of Professor Michler, developed a stand-alone algorithm for finding short-word generators. Paul K. Young, a current graduate student of the Mathematics Department of Cornell University, polished the idea and implemented the algorithm in MAGMA. Since Young's implementation had some problems when dealing with groups of large order (or permutation groups of large degree), the author modified and added several new ideas. Thus the author produced a relatively efficient implementation of the resulting algorithm.

Besides this short-word-generator algorithm, another algorithm for getting a short word for an element of a group with given generators is also presented in the first section of this thesis. For each algorithm, the description and its MAGMA implementation are given.

These new algorithms are indeed crucial in the author's construction of Fi_{23} ; they are not just for documentation. They provide a successful method for constructing the 2-fold cover 2Fi_{22} where Holt's Algorithm failed.

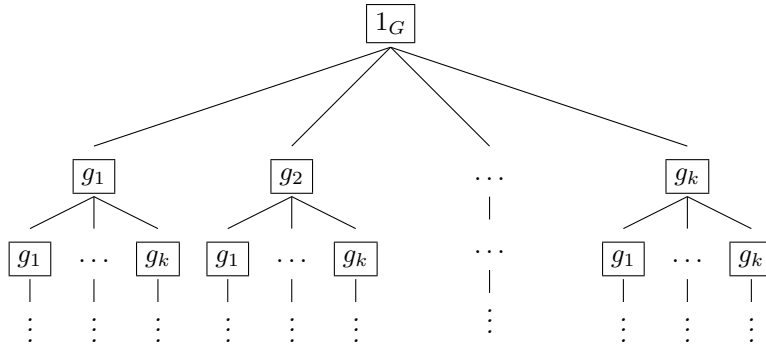
For readers who would like to find more background materials for this thesis, they should see Holt's book [7] for computational group theory, and Michler's book [12] for algorithmic representation theory of finite simple groups.

1. ALGORITHMS

In this section, a finite group G is always assumed to be realized as a permutation group or a matrix group. This allows us to compute the orders of elements and subgroups of G , and check equalities.

Before going into the actual algorithms, it is natural to have definitions of some vocabulary: ‘word-tree’, ‘length of a word’, ‘short-word’, ‘lexicographic order of words’, and so on. The following definition, as stated in Definition 5.3.9 of [12], is due to M. Kratzer.

Definition 1.1 (Word Tree, Word Length). *Let G be a finite group. Let $\{g_1, g_2, \dots, g_k\}$ be a fixed set of generators for G . The infinitely deep k -nary tree $\mathcal{C}(G)$ in which the root vertex is marked by the identity $1_G \in G$ and the k successors of each vertex are marked successively by g_1, g_2, \dots, g_k is called the “complete word tree of G ”:*



There is a canonical one-to-one correspondence between vertices in $\mathcal{C}(G)$ and words in generators of G : For any vertex v of $\mathcal{C}(G)$ let w_v denote the incremental product of the vertex markers occurring along the direct path from the root vertex to v in $\mathcal{C}(G)$. Conversely, starting from the root vertex and reading a given word $w = w(g_1, g_2, \dots, g_k)$ in generators of G like a sequence of directions guides one to the unique vertex v_w in $\mathcal{C}(G)$ such that $w_{v_w} = w$.

The “length” of a word w is the depth of the unique vertex v_w corresponding to w in the word tree $\mathcal{C}(G)$, i.e. the number of steps needed to reach the vertex v_w from root vertex. For example, the word $g_1^2 g_3 g_2$ has length 4.

Now, a “short-word” refers to a word of short length, though notion of “short” might not be consistent. To simplify arguments, we may look only at the indices of the generators. That is, $g_1^2 g_3 g_2$ can be identified with $[1, 1, 3, 2]$. Precise definition of this idea is as follows:

Definition 1.2 (Numerical Word). *A “numerical word of k generators” is a finite sequence (can be empty sequence) of integers in $\{1, 2, \dots, k\}$. For example, $[5, 1, 2, 2, 2, 3, 4, 4, 5]$ is an example of a numerical word of 5 generators, as well as a numerical word of 6 generators, but not a numerical word of 4 generators.*

Let $\{g_1, g_2, \dots, g_k\}$ be a fixed set of generators of a finite group G . Then, if we identify each g_i with i , there is a natural one-to-one correspondence between the set of all numerical word of k generators and $\mathcal{C}(G)$. For example, the word $g_1 g_4^3 g_2 g_3^2$ corresponds to the numerical word $[1, 4, 4, 4, 2, 3, 3]$.

Now we can define a total ordering on $\mathcal{C}(G)$, i.e. on the set of all finite words in the given (ordered) generators g_1, g_2, \dots, g_k of G .

Definition 1.3 (Lexicographical Order). *Let $\{g_1, g_2, \dots, g_k\}$ be a fixed set of generators of a finite group G . Let v_u and v_w be the vertices of $\mathcal{C}(G)$, corresponding to distinct words u and w in g_1, g_2, \dots, g_n , respectively. Let N_u and N_w be the numerical words of k generators corresponding to v_u and v_w , respectively (each g_i is identified with i). Then, $u < w$ if and only if one of the followings hold (this is called “lexicographic order”):*

- (1) *length of $u <$ length of w*
- (2) *length of $u =$ length of w , and $N_u[n] < N_w[n]$ holds, where n is the smallest number such that $N_u[n] \neq N_w[n]$.*

Remark 1.4. *If u and w are distinct words of G (in given generators), then exactly one of $u < w$ or $w < u$ holds.*

The main algorithm is named `GetShortGens`; for any finite group G with given set of generators, for any subgroup S of G , this algorithm returns a short-word generating set for S in terms of the given generators. It needs three other small programs, named `EnumWords`, `ReduceGensForGroup` and `Word2Elt`. All four programs are described below, with description of the algorithms and their implementation in MAGMA. The first algorithm `EnumWords` enables us to descend one level deeper in the word tree:

Algorithm 1.5 (`EnumWords`). *Let k be the number of given generators for the finite group of interest. Let W be a sequence of pre-built numerical words in k generators. Let s, e be integers s.t. $1 \leq s \leq e \leq \#W$ (s for “start”, e for “end”). Then, we can get a new sequence W_1 of numerical words by adding new words to W , where the new words are obtained by appending $1, 2, \dots, k$ at the end of the words $W[s], W[s+1], \dots, W[e]$.*

Implementation 1.6 (`EnumWords`).

function `EnumWords(W, st_end, nGens)`

```

local NewW, Newst_end;

NewW := W;
if #NewW eq 0 then
    NewW := [[i] : i in [1..nGens]];
    Newst_end := [1, nGens];
else
    for i:=st_end[1] to st_end[2] do
        for j:=1 to nGens do
            Append(~NewW, Append(W[i],j));
        end for;
    end for;
    Newst_end := [st_end[2]+1, #NewW];
end if;
return NewW, Newst_end;

end function;
```

Example 1.7 (`EnumWords`). *MAGMA code*

```

> W:=[ [1,3,4], [3,3,2], [2,1], [4] ];
> s:=2; e:=3;
> W:=EnumWords(W, [s,e], 4);
> W;
[ [1,3,4], [3,3,2], [2,1], [4], [3,3,2,1], [3,3,2,2], [3,3,2,3],
  [3,3,2,4], [2,1,1], [2,1,2], [2,1,3], [2,1,4] ]
```

The next algorithm `ReduceGensForGroup` gives a method to reduce the given set of generators as much as possible, while our subgroup of interest still should lie inside the subgroup generated by the reduced set of generators:

Algorithm 1.8 (`ReduceGensForGroup`). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any subgroup S of G , a subset of $\{g_1, g_2, \dots, g_k\}$ which generates a subgroup of G containing S is obtained by the following method:*

[Reducing original generators] If there is some $i \in \{1, 2, \dots, k\}$ such that the set $\{g_1, \dots, g_{i-1}, g_{i+1}, \dots, g_k\}$ generates a subgroup of G containing S , pick the largest such i . Now, call this function `ReduceGensForGroup` recursively, with same G and S , with reduced set of generators $g_1, \dots, g_{i-1}, g_{i+1}, \dots, g_k$ (with corresponding names). If there is no such i , then return the original set g_1, g_2, \dots, g_k , with their names.

Implementation 1.9 (`ReduceGensForGroup`).

```
function ReduceGensForGroup(G, Target : wordgens:=[], gencollection:=[1..#Generators(G)],
exclude:=sub⟨G⟩, CoverGroup:=G)
```

```
  local reducedlist;
```

```
  if #wordgens eq 0 or #Generators(G) ne #wordgens then
    wordgens:=[("$." cat Sprint(i)) : i in [1..#Generators(G)]];
  end if;
  for i:=1 to #Generators(G) do
    reducedlist := Exclude([1..#Generators(G)], #Generators(G)+1-i);
    if (Target meet sub⟨CoverGroup|[G.j : j in reducedlist], exclude) eq Target then
      return ReduceGensForGroup(sub⟨G|[G.j : j in reducedlist]), Target :
wordgens:=[wordgens[j] : j in reducedlist], gencollection:=[gencollection[j] : j in
reducedlist], exclude:=exclude, CoverGroup:=CoverGroup);
    end if;
  end for;
  return G, wordgens, gencollection;
```

```
end function;
```

The following algorithm `Word2Elt` converts a numerical word to the corresponding actual element of the group:

Algorithm 1.10 (`Word2Elt`). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any numerical word w in k generators, return the element in G corresponding to the word w by the following steps:*

Step 1 *Let $e = 1$, the identity element of G .*

Step 2 *If w is an empty word, return e . If not, and if w can be written as $w = [a_1, a_2, \dots, a_m]$ where $a_i \in \{1, 2, \dots, k\}$, then let $e := e \cdot g_{a_1}$.*

Step 3 *Let $w := [a_2, a_3, \dots, a_m]$, and go to **Step 2**.*

Implementation 1.11 (`Word2Elt`).

```
function Word2Elt(G, word)
```

```
  local elt;
```

```
  elt := Id(G);
  for i:=1 to #word do
    elt := elt * G.word[i];
  end for;
  return elt;
```

```
end function;
```

Example 1.12 (Word2Elt). MAGMA code (Note: MAGMA composes two permutations from left, not from right)

```
> G:=sub<Sym(3)| Sym(3)!(1,2), Sym(3)!(1,2,3)>;
> Word2Elt(G, [1,2,2]);
(2,3)
```

Finally, the next algorithm `GetShortGens` enables us to obtain a short-word generating set for a subgroup of a group with given generators:

Algorithm 1.13 (GetShortGens). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any subgroup S of G , a generating set of S consisting of short-word elements of G in terms of g_1, g_2, \dots, g_k is obtained by the following steps:*

Step 1 [Reducing original generators] *If desired, try to get a subset of $\{g_1, g_2, \dots, g_k\}$ which generates a subgroup of G containing S , using the command `ReduceGensForGroup`. For convenience, suppose that the set of generators g_1, g_2, \dots, g_k is already a result of this reducing process.*

Step 2 [Building word list, and finding generators]

- (1) Set $F = \langle 1 \rangle$ (trivial subgroup).
- (2) Let $W = [[1], [2], \dots, [k]]$, the set of words, initially set to have only simplest numerical words of k generators of length 1.
- (3) Take the word w in W of lowest lexicographic order which is not checked yet, and let $x = \text{Word2Elt}(G, w)$, the element of G corresponding to the word w . If there is some $m \in \{1, 2, \dots, \text{Order}(w) - 1\}$ such that $w^m \in S$ and $w^m \notin F$, then enlarge F by $F := \langle F, w^m \rangle$.
- (4) If $F = S$, then proceed to **Step 3**. If not, enlarge W by appending $1, 2, \dots, k$ to all words of W of longest length, using the command `EnumWords` (this is same as descending one level deeper in the word tree). Now, go to (3).

Step 3 [Printing] *Print the words obtained.*

Implementation 1.14 (GetShortGens).

```
function GetShortGens(G, Target : exclude := sub(G), limit:=0, wordgens:=[], Words:=[],
startpoint:=1, powers:=:[1], Hard:=true, OrderRestriction:=[], CoverGroup:=G,
generatecheck:=true, auto:=true, EltReturn:=false, ReduceMore:=true)
```

```
local gens, iter, st.end, tempelt, tempord, temppow, WordsSoFar, WordsNumSoFar,
gencollection, SubgroupSoFar, generatingset;
```

```
gens := Generators(G);
```

```
function GetGeneratingSet(WordsForGenerators)
return [Word2Elt(G, WordsForGenerators[i][1])^WordsForGenerators[i][2] :
i in [1..#WordsForGenerators]];
end function;
```

```
if generatecheck and Target meet sub(CoverGroup|G,exclude) ne Target then
print "can't generate subgroup";
return "";
end if;
```

```
if #wordgens eq 0 or #Generators(G) ne #wordgens then
if #wordgens ne 0 and #Generators(G) ne #wordgens then
print "the number of generator names you provided is incompatible
with the number of generators of the group, so re-building names";
end if;
wordgens:=["$" cat Sprint(i) :i in [1..#Generators(G)]];
```

```

end if;

if Hard then
  print "Reducing Generators...";
  time G, wordgens, gencollection := ReduceGensForGroup(G, Target :
wordgens:=wordgens,exclude:=exclude,CoverGroup:=CoverGroup);
  printf "Using only %o generators %o, out of %o ", #Generators(G),wordgens,#gens;
  gens := Generators(G);
else
  gencollection := [1..#Generators(G)];
end if;

Include(~powers,1);
Sort(~powers);

WordsSoFar:=[];
WordsNumSoFar:=[];
SubgroupSoFar := Target meet exclude;
iter:=0;

while limit eq 0 or iter lt limit do
  if #gens eq 1 and iter ge Order(G.1) then
    print "use other method";
    return "",[];
  end if;

  iter:=iter+1;
  printf "%o-th iteration\n",iter;
  if #Words eq 0 then
    Words, st.end := EnumWords([], [1,1], #gens);
    startpoint:=1;
  end if;

  for j:=startpoint to #Words do
    if #OrderRestriction eq 0 then
      tempelt := Word2Elt(G, Words[j]);
      if auto then powers:=[1..(Order(tempelt)-1)]; end if;
      if exists(tempelt){x : x in powers | tempelt^x notin SubgroupSoFar
and tempelt^x in Target} then
        Append(~WordsSoFar, WordPrint(Words[j], wordgens:power:=tempelt));
        Append(~WordsNumSoFar, <[gencollection[Words[j][k]]:k in [1..#Words[j]]],
tempelt);
        printf "Got a new elt: %o\n", WordsSoFar[#WordsSoFar];
        SubgroupSoFar := sub<Target|SubgroupSoFar, Word2Elt(G,Words[j])^tempelt>;
        if SubgroupSoFar eq Target then
          printf "Got a generating set.\n";
          if ReduceMore then
            print "Getting a smaller generating set...";
            time G, wordgens, gencollection :=
ReduceGensForGroup(sub<G|GetGeneratingSet(WordsNumSoFar)), Target :
wordgens:=[],exclude:=exclude,CoverGroup:=CoverGroup);
            printf "Resulting set has %o generators, out of %o\n", #wordgens,#WordsSoFar;
            printf "subcollection indices:%o\n",gencollection;
            gens := Generators(G);
          end if;

          WordsSoFar := [WordsSoFar[gencollection[i]] : i in [1..#gencollection]];
          WordsNumSoFar := [WordsNumSoFar[gencollection[i]] : i in [1..#gencollection]];
          if not EltReturn then
            return WordsSoFar, WordsNumSoFar;
          else
            return WordsSoFar, WordsNumSoFar, GetGeneratingSet(WordsNumSoFar);
          end if;
        end if;
      end if;
    end if;
  end if;
end if;

```

```

        end if;
    end if;
else // if there is some restriction on orders
    tempelt := Word2Elt(G, Words[j]);
    tempord := Order(tempelt);
    for ord in OrderRestriction do
        if tempord mod ord eq 0 then
            temppow := Integers()!(tempord/ord);
            if tempelt^tempow notin SubgroupSoFar and tempelt^tempow in Target then
                Append(~WordsSoFar, WordPrint(Words[j],wordgens:power:=tempow));
                Append(~WordsNumSoFar, <[gencollection[Words[j][k]:k in
[1..#Words[j]]],tempow]>);
                printf "Got a new elt of order %o: %o\n",ord,WordsSoFar[#WordsSoFar];
                SubgroupSoFar := sub(Target|SubgroupSoFar, Word2Elt(G,Words[j])^tempow);
                if SubgroupSoFar eq Target then
                    printf "Done.\n";
                    if not EltReturn then
                        return WordsSoFar, WordsNumSoFar;
                    else
                        return WordsSoFar, WordsNumSoFar, Generators(SubgroupSoFar);
                    end if;
                end if;
            end if;
        end if;
    end for;
end for;
Words, st_end := EnumWords(Words, st_end, #gens);
startpoint := st_end[1];
end while;

print "Couldn't generate group";
return WordsSoFar;
end function;

```

Example 1.15 (GetShortGens). MAGMA code

```

> g1:=Sym(8)!(1,2); g2:=Sym(8)!(1,2,3,4,5,6,7,8);
> G:=sub<Sym(8)|g1,g2>;
> S := sub<Sym(8)| Sym(8)!(1,3,6)(2,4), Sym(8)!(1,7,8)(2,5)>;
> res := GetShortGens(G,S : wordgens=['g1','g2']);
> res;
[ (g2*g1*g2^4)^5, (g1*g2*g1*g2^4)^3, (g1*g2^3*g1*g2*g1)^2,
(g1*g2*g1*g2*g1*g2^3*g1)^3 ]

```

For a given element of a finite group with a given generating set, it is often important to obtain a short word for the element in terms of given generating set. This element-version (as opposed to subgroup-version: `GetShortGens`) of short-word program is named `LookupWord`, and it needs a different version of `ReduceGensForGroup` which is called `ReduceGensForElt`. It is designed for finding a word of an element; it reduces the given set of generators as much as possible, while the element of interest still should lie inside the subgroup generated by the reduced set of generators:

Algorithm 1.16 (`ReduceGensForElt`). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any element x of G , a subset of $\{g_1, g_2, \dots, g_k\}$ which generates a subgroup of G containing S is obtained by the following method:*

[Reducing original generators] If there is some $i \in \{1, 2, \dots, k\}$ such that the set $\{g_1, \dots, g_{i-1}, g_{i+1}, \dots, g_k\}$ generates a subgroup of G containing S , pick the largest such i . Now, call this function `ReduceGensForElt` recursively, with same G and S ,

with reduced set of generators $g_1, \dots, g_{i-1}, g_{i+1}, \dots, g_k$ (with corresponding names). If there is no such i , then return the original set g_1, g_2, \dots, g_k , with their names.

Implementation 1.17 (ReduceGensForElt).

```
function ReduceGensForElt(G, TargetElt : wordgens:=[], gencollection:=[1..#Generators(G)])
local reducedlist;

for i:=1 to #Generators(G) do
  reducedlist := Exclude([1..#Generators(G)],#Generators(G)+1-i);
  if TargetElt in sub⟨G|[G.j : j in reducedlist]⟩ then
    return ReduceGensForElt(sub⟨G|[G.j : j in reducedlist]⟩, TargetElt :
wordgens:=reducedlist, gencollection:=[gencollection[j]:j in reducedlist]);
  end if;
end for;
return G, wordgens, gencollection;
end function;
```

The algorithm `LookupWord` enables us to obtain a short word for an element of a group with given generators:

Algorithm 1.18 (`LookupWord`). Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any element x of G , a short word for x in terms of g_1, g_2, \dots, g_k is obtained by the following steps:

Step 1 [*Reducing original generators*] If desired, try to get a subset of $\{g_1, g_2, \dots, g_k\}$ which generates a subgroup of G containing x , using the command `ReduceGensForElt`. For convenience, suppose that the set of generators g_1, g_2, \dots, g_k is already a result of this reducing process.

Step 2 [*Building word list, and finding generators*]

- (1) Let $W = [[1], [2], \dots, [k]]$, the set of words, initially set to have only simplest numerical words of k generators of length 1.
- (2) Take the word w in W of lowest lexicographic order which is not checked yet, and let $y = \text{Word2Elt}(G, w)$, the element of G corresponding to the word w .
- (3) Let $r = \text{Order}(y)$ and $s = \text{Order}(x)$. If $s \nmid r$, go to (5).
- (4) If there is some $m \in \{t \in \mathbb{Z} \mid 1 \leq t < s, \gcd(t, \text{Order}(x)) = 1\}$ such that $w^{rm/s} = x$, then proceed to **Step 3**.
- (5) Enlarge W by appending $1, 2, \dots, k$ to all words of W of longest length, using the command `EnumWords` (this is same as descending one level deeper in the word tree). Now, go to (2).

Step 3 [*Printing*] Print the words obtained.

Alternative option : The above algorithm finds a short word which is equal to the given element. A similar algorithm can be used to find a short word which is conjugate to the given element: in **Step 2**(4), we look for m such that $w^{rm/s}$ is conjugate to x (the equality test is replaced by the conjugacy test). This option is incorporated in the following implementation, as the hidden parameter `ConjugateCheck`; if we set `ConjugateCheck:=true`, then the following program finds a short word which is conjugate to the given element.

Implementation 1.19 (`LookupWord`).

```
function LookupWord(G, TargetElt : limit:=0, wordgens:=[], Words:=[], st_end:=[],
startpoint:=1, Hard:=true, ConjugateCheck:=false, CoverGroup:=G, containcheck:=true,
InfoLevel:=2)
```

```
local Ord, gens, iter;
```

```

local tempelt, tempord, tempow;

Ord := Order(TargetElt);
gens := Generators(G);

if containcheck and TargetElt notin G then
  print "Couldn't find a word for it";
  return "";
end if;

if TargetElt eq Id(G) then
  if InfoLevel gt 1 then print "it is the identity!"; end if;
  return "Id($)", <[],1>;
end if;

if #wordgens eq 0 or #gens ne #wordgens then
  for i:=1 to #gens do
    Append(~wordgens, "$." cat IntegerToString(i));
  end for;
end if;

if Hard and not ConjugateCheck then
  print "Reducing Generators...";
  time G, wordgens, gencollection := ReduceGensForElt(G, TargetElt : wordgens:=wordgens);
  printf "Using only %o generators %o, out of %o ", #Generators(G),wordgens,#gens;
  gens := Generators(G);
else
  gencollection := [1..#Generators(G)];
  if #wordgens eq 0 or #Generators(G) ne #wordgens then
    wordgens:=[("$." cat Sprint(i)) :i in [1..#Generators(G)]];
  end if;
end if;

iter:=0;
while limit eq 0 or iter lt limit do
  iter:=iter+1;
  if InfoLevel gt 1 then
    printf "%o-th iteration\n",iter;
  end if;
  if #Words eq 0 or #st_end eq 0 then
    Words, st_end := EnumWords([], [1,1], #gens);
    startpoint:=1;
  end if;

  for j:=startpoint to #Words do
    tempelt := Word2Elt(G, Words[j]);
    tempord := Order(tempelt);
    if tempord mod Ord eq 0 then
      tempow := Integers()!(tempord/Ord);
      for addi in [a:a in [1..Ord]|GCD(a,Ord) eq 1] do
        if tempelt^(tempow*addi) eq TargetElt then
          if InfoLevel gt 1 then print "Got a word (exact)"; end if;
          return WordPrint(Words[j], wordgens:power:=(tempow*addi)),
<[gencollection[Words[j][k]:k in [1..#Words[j]]], tempow*addi];
          elif ConjugateCheck and IsConjugate(CoverGroup,tempelt^(tempow*addi),TargetElt)
then
            if InfoLevel gt 1 then print "Got a conjugate word (new ver)"; end if;
            return WordPrint(Words[j], wordgens:power:=(tempow*addi)), <[Words[j][k]:k in
[1..#Words[j]]], tempow*addi);
          end if;
        end for;
      end if;
    end if;
  end for;
end for;

```

```

Words, st_end := EnumWords(Words, st_end, #gens);
startpoint := st_end[1];
end while;

print "Couldn't find a word";
return "";

end function;

Example 1.20 (LookupWord). MAGMA code
> g1:=Sym(8)!(1,2); g2:=Sym(8)!(1,2,3,4,5,6,7,8);
> G:=sub<Sym(8)|g1,g2>;
> x := Sym(8)!(2, 8, 7, 6, 4, 3);
> res := LookupWord(G,x : wordgens:=['g1','g2']);
> res;
g1*g2^4*g1*g2^3

```

Sometimes, the subgroup or the element that we want to get short words of lies too deep inside the group (whose generators are given), so `GetShortGens` or `LookupWord` doesn't work well. For example, if an element x can't be represented by a word of length ≤ 24 in terms of given generators, then `LookupWord` either takes too long a time to get the word for x , or causes memory overflow due to too much required space for all the words built up so far. The author found a strategy to overcome this issue, described as follows.

The strategy needs a proper subgroup T of G , which contains the target subgroup S or the target element x . A standard trick to get such T is to take the normalizer of S in G , the centralizer of x in G , or the normalizer of $\langle x \rangle$ of G .

Strategy 1.21 (two-step `GetShortGens`). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any subgroups S and T of G such that $S \leq T \leq G$, a short-word generating set of S in terms of g_1, g_2, \dots, g_k is obtained by the following steps:*

Step 1 [*Generators for T*] *Get short-word generators t_1, t_2, \dots, t_n for T in terms of g_1, g_2, \dots, g_k using `GetShortGens`.*

Step 2 [*Generators for S*] *Get short-word generators for S in terms of t_1, t_2, \dots, t_n using `GetShortGens`.*

Strategy 1.22 (two-step `LookupWord`). *Let G be a finite group generated by the given generators g_1, g_2, \dots, g_k . For any element $x \in G$ and a subgroup T of G such that $x \in T \leq G$, a short word for x in terms of g_1, g_2, \dots, g_k is obtained by the following steps:*

Step 1 [*Generators for T*] *Get short-word generators t_1, t_2, \dots, t_n for T in terms of g_1, g_2, \dots, g_k using `GetShortGens`.*

Step 2 [*Short Words for x*] *Get short word for x in terms of t_1, t_2, \dots, t_n using `LookupWord`.*

The above two strategies are used throughout this thesis and therefore constantly referred to. Note that sometimes it can be helpful to iterate the strategies many times. For example, for a subgroup S of G , it can be a good idea to try to find subgroups T_1 and T_2 such that $S \leq T_1 \leq T_2 \leq G$ and then find short-word generators for T_2 , T_1 , and finally S in turn.

2. MICHLER'S ALGORITHM

In this section, G. Michler's Algorithm 2.5 of [11] is presented. This algorithm gives a uniform method to construct all finite simple groups (not having Sylow 2-subgroups which are cyclic, dihedral or semi-dihedral), from indecomposable subgroups of $\text{GL}_n(2)$. The author's algorithms described in the previous section are used to implement this algorithm of Michler. In particular, in this thesis, Fi_{23} is constructed by the following algorithm.

Algorithm 2.1. *Let T be an indecomposable subgroup of $\text{GL}_n(2)$ acting on $V = F^n$ by matrix multiplication.*

- Step 1: *Calculate a faithful permutation representation PT of T and a finite presentation $T = \langle t_i \mid 1 \leq i \leq r \rangle$ with set $\mathcal{R}(T)$ of defining relations.*
- Step 2: *Compute all extension groups E of T by V by means of Holt's Algorithm [7]. Determine a complete set \mathfrak{S} of non isomorphic extension groups E by means of the Cannon-Holt Algorithm [14].*
- Step 3: *Let $E \in \mathfrak{S}$. From the given presentation of E determine a faithful permutation representation PE of E . Using it and Kratzer's Algorithm 5.3.18 of [12] calculate a complete system of representatives of all the conjugacy classes of E .*
- Step 4: *Let $z \neq 1$ be a 2-central involution of E . Calculate $D = C_E(z)$ and fix a Sylow 2-subgroup S of D . Check that the elementary abelian normal subgroup V of E is a maximal elementary abelian normal subgroup of S . If it is not maximal, then the algorithm terminates.*
- Step 5: *Construct a group $H > D$ with the following properties:*
 - (a) *z belongs to the center $Z(H)$ of H .*
 - (b) *The index $|H : D|$ is odd.*
 - (c) *The normalizer $N_H(V) = D = C_E(z)$.*

If no such H exists the algorithm terminates.

Otherwise, apply for each constructed group H the following steps of Algorithm 7.4.8 of [12]. By step 5(c) it may be assumed from now on that $D = H \cap E$.

After the Step 5, the remaining steps are identical to Algorithm 7.4.8 of [12], so omitted here.

3. EXTENSIONS OF MATHIEU GROUP M_{23}

The Mathieu group M_{23} is defined in Definition 8.2.1 of [12] by means of generators and relations. This beautiful presentation is due to J.A. Todd. The irreducible 2-modular representations of the Mathieu group M_{23} were determined by G. James [8]. Here only the 2 non isomorphic simple modules V_i , $i = 1, 2$, of dimension 11 over $F = \text{GF}(2)$ will be considered. Todd's permutation representations of the Mathieu groups are stated in Lemma 8.2.2 of [12]. Therefore all conditions of Holt's Algorithm [7] implemented in MAGMA are satisfied. It is applied here. Thus it is shown in this section that for the simple module V_1 there are exactly two extensions of M_{23} by V_1 , the split extension E_1 and the non-split extension E and that M_{23} has only the split extension E_2 by V_2 . It will be shown that the applications of Algorithm 2.1 to E_1 and E_2 fail. Therefore only the constructed presentation of E is given in Lemma 3.1.

Lemma 3.1. *Let $\mathcal{M}_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$ be the finitely presented group with set of defining relations $\mathcal{R}(\mathcal{M}_{23})$ given in Definition 8.2.1 of [12]. Then the following statements hold:*

- (a) *A faithful permutation representation of degree 23 of \mathcal{M}_{23} is stated in Lemma 8.2.2 of [12].*
- (b) *The first irreducible representation (ρ_1, V_1) of \mathcal{M}_{23} is described by the following matrices:*

$$\rho_1(a) = \begin{pmatrix} 010011111100 \\ 011000111111 \\ 000000100000 \\ 00110111010 \\ 11000111011 \\ 000001000000 \\ 001000000000 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix}, \quad \rho_1(b) = \begin{pmatrix} 010000000000 \\ 100000000000 \\ 000010000000 \\ 11010101110 \\ 001000000000 \\ 000001000000 \\ 11000110011 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix},$$

$$\rho_1(c) = \begin{pmatrix} 000010000000 \\ 001000000000 \\ 010000000000 \\ 10011101001 \\ 100000000000 \\ 000001000000 \\ 01100011111 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix}, \quad \rho_1(d) = \begin{pmatrix} 01111010001 \\ 10111011100 \\ 00110111010 \\ 00000010000 \\ 00011110111 \\ 00000100000 \\ 00010000000 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix},$$

$$\rho_1(t) = \begin{pmatrix} 01000000100 \\ 10011101101 \\ 01001111000 \\ 10111011000 \\ 01111010101 \\ 00000000100 \\ 11010101010 \\ 00000001100 \\ 00000100100 \\ 00000000110 \\ 00000000101 \end{pmatrix}, \quad \rho_1(g) = \begin{pmatrix} 01010000000 \\ 10010000000 \\ 01011111000 \\ 00010000000 \\ 11010110011 \\ 10101011100 \\ 01110011111 \\ 10100000111 \\ 00001110111 \\ 00010000010 \\ 00010000001 \end{pmatrix},$$

$$\rho_1(h) = \begin{pmatrix} 10000100000 \\ 10011001001 \\ 00110011010 \\ 00000110000 \\ 11000010011 \\ 00000100000 \\ 00010100000 \\ 00000100010 \\ 00000100100 \\ 00000101000 \\ 00000100001 \end{pmatrix}, \quad \rho_1(i) = \begin{pmatrix} 10011001001 \\ 01000100000 \\ 11010000110 \\ 00010100000 \\ 00001100000 \\ 00000100000 \\ 01001011100 \\ 00000101000 \\ 00000100100 \\ 00000100001 \\ 00000100001 \end{pmatrix},$$

$$\rho_1(j) = \begin{pmatrix} 01000000000 \\ 10000000000 \\ 00001000000 \\ 00010000000 \\ 00100000000 \\ 00000000000 \\ 00000000100 \\ 01100011111 \\ 00000001000 \\ 00000100000 \\ 00000000010 \\ 11101100101 \end{pmatrix}.$$

- (c) *The second irreducible representation (ρ_2, V_2) of \mathcal{M}_{23} is described by the transpose inverse matrices of the generating matrices of \mathcal{M}_{23} defining (ρ_1, V_1) :*

$$\begin{aligned}\rho_2(a) &= [\rho_1(a)^{-1}]^T, \quad \rho_2(b) = [\rho_1(b)^{-1}]^T, \quad \rho_2(c) = [\rho_1(c)^{-1}]^T, \\ \rho_2(d) &= [\rho_1(d)^{-1}]^T, \quad \rho_2(t) = [\rho_1(t)^{-1}]^T, \quad \rho_2(g) = [\rho_1(g)^{-1}]^T, \\ \rho_2(h) &= [\rho_1(h)^{-1}]^T, \quad \rho_2(i) = [\rho_1(i)^{-1}]^T, \quad \text{and } \rho_2(j) = [\rho_1(j)^{-1}]^T.\end{aligned}$$

- (d) $\dim_F[H^2(\mathcal{M}_{23}, V_1)] = 1$ and $\dim_F[H^2(\mathcal{M}_{23}, V_2)] = 0$.
 (e) *The presentations of the split extensions E_1 and E_2 of \mathcal{M}_{23} by V_1 and V_2 , respectively, can be constructed from the matrices of (b) and (c) using MAGMA.*
 (f) *The unique non-split extension E of \mathcal{M}_{23} by V_1 has the presentation*

$$E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1, v_i \mid 1 \leq i \leq 11 \rangle$$

with set $\mathcal{R}(E)$ of defining relations consisting of the following set of relations:

$$\begin{aligned}a_1^2 &= b_1^2 = c_1^2 = d_1^2 = t_1^3 = g_1^4 = h_1^4 = j_1^2 = 1, \\ (b_1, a_1) &= (c_1, a_1) = (d_1, a_1) = (h_1, a_1) = (c_1, b_1) = (d_1, b_1) = (d_1, c_1) = 1, \\ (j_1, b_1) &= (j_1, c_1) = (j_1, g_1) = 1, \\ v_r^2 &= 1 \quad \text{for } 1 \leq r \leq 11, \\ (v_r, v_s) &= 1 \quad \text{for } 1 \leq r, s \leq 11, \\ a_1^{-1}v_1a_1v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1} &= a_1^{-1}v_2a_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\ a_1^{-1}v_3a_1v_7^{-1} &= a_1^{-1}v_7a_1v_3^{-1} = a_1^{-1}v_4a_1v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1, \\ a_1^{-1}v_5a_1v_1^{-1}v_2^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\ (a_1, v_6) &= (a_1, v_8) = (a_1, v_9) = (a_1, v_{10}) = (a_1, v_{11}) = 1, \\ b_1^{-1}v_1b_1v_2^{-1} &= b_1^{-1}v_2b_1v_1^{-1} = b_1^{-1}v_3b_1v_5^{-1} = b_1^{-1}v_5b_1v_3^{-1} = 1, \\ b_1^{-1}v_4b_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} &= 1, \\ (b_1, v_6^{-1}) &= (b_1, v_8^{-1}) = (b_1, v_9^{-1}) = (b_1, v_{10}^{-1}) = (b_1, v_{11}^{-1}) = 1, \\ b_1^{-1}v_7b_1v_1^{-1}v_2^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} &= c_1^{-1}v_1c_1v_5^{-1} = c_1^{-1}v_2c_1v_3^{-1} = 1, \\ c_1^{-1}v_3c_1v_2^{-1} &= c_1^{-1}v_5c_1v_1^{-1} = c_1^{-1}v_4c_1v_1^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{11}^{-1} = 1, \\ (c_1, v_6^{-1}) &= (c_1, v_8^{-1}) = (c_1, v_9^{-1}) = (c_1, v_{10}^{-1}) = (c_1, v_{11}^{-1}) = 1, \\ c_1^{-1}v_7c_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= d_1^{-1}v_1d_1v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_{11}^{-1} = 1, \\ d_1^{-1}v_2d_1v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1} &= d_1^{-1}v_3d_1v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1, \\ d_1^{-1}v_4d_1v_7^{-1} &= d_1^{-1}v_7d_1v_4^{-1} = d_1^{-1}v_5d_1v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\ (d_1, v_6^{-1}) &= (d_1, v_8^{-1}) = (d_1, v_9^{-1}) = (d_1, v_{10}^{-1}) = (d_1, v_{11}^{-1}) = 1, \\ t_1^{-1}v_1t_1v_2^{-1}v_9^{-1} &= t_1^{-1}v_6t_1v_9^{-1} = t_1^{-1}v_2t_1v_1^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1, \\ t_1^{-1}v_3t_1v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1} &= t_1^{-1}v_4t_1v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1} = 1, \\ t_1^{-1}v_5t_1v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{11}^{-1} &= t_1^{-1}v_7t_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_{10}^{-1} = 1, \\ t_1^{-1}v_8t_1v_8^{-1}v_9^{-1} &= t_1^{-1}v_9t_1v_6^{-1}v_9^{-1} = t_1^{-1}v_{10}t_1v_9^{-1}v_{10}^{-1} = t_1^{-1}v_{11}t_1v_9^{-1}v_{11}^{-1} = 1, \\ g_1^{-1}v_1g_1v_2^{-1}v_4^{-1} &= g_1^{-1}v_2g_1v_1^{-1}v_4^{-1} = g_1^{-1}v_3g_1v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1} = 1,\end{aligned}$$

$$\begin{aligned}
(g_1, v_4) &= g_1^{-1}v_5g_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
g_1^{-1}v_6g_1v_1^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1} &= g_1^{-1}v_7g_1v_2^{-1}v_3^{-1}v_4^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
g_1^{-1}v_8g_1v_1^{-1}v_3^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= g_1^{-1}v_9g_1v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
g_1^{-1}v_{10}g_1v_4^{-1}v_{10}^{-1} &= g_1^{-1}v_{11}g_1v_4^{-1}v_{11}^{-1} = h_1^{-1}v_1h_1v_1^{-1}v_6^{-1} = 1, \\
h_1^{-1}v_4h_1v_6^{-1}v_7^{-1} &= (h_1, v_6) = h_1^{-1}v_2h_1v_1^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{11}^{-1} = 1, \\
h_1^{-1}v_3h_1v_3^{-1}v_4^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} &= h_1^{-1}v_5h_1v_1^{-1}v_2^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
h_1^{-1}v_7h_1v_4^{-1}v_6^{-1} &= h_1^{-1}v_8h_1v_6^{-1}v_{10}^{-1} = h_1^{-1}v_9h_1v_6^{-1}v_9^{-1} = 1, \\
h_1^{-1}v_{10}h_1v_6^{-1}v_8^{-1} &= h_1^{-1}v_{11}h_1v_6^{-1}v_{11}^{-1} = i_1^{-1}v_1i_1v_1^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{11}^{-1} = 1, \\
i_1^{-1}v_2i_1v_2^{-1}v_6^{-1} &= i_1^{-1}v_4i_1v_4^{-1}v_6^{-1} = i_1^{-1}v_5i_1v_5^{-1}v_6^{-1} = 1, \\
i_1^{-1}v_3i_1v_1^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} &= 1, \quad (i_1, v_6) = 1, \\
i_1^{-1}v_7i_1v_2^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1} &= i_1^{-1}v_{11}i_1v_6^{-1}v_{11}^{-1} = i_1^{-1}v_8i_1v_6^{-1}v_8^{-1} = 1, \\
i_1^{-1}v_9i_1v_6^{-1}v_9^{-1} &= i_1^{-1}v_{10}i_1v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
j_1^{-1}v_1j_1v_2^{-1} &= j_1^{-1}v_2j_1v_1^{-1} = j_1^{-1}v_3j_1v_5^{-1} = (j_1, v_4) = (j_1, v_8) = 1, \\
(j_1, v_{10}) &= j_1^{-1}v_5j_1v_3^{-1} = j_1^{-1}v_6j_1v_9^{-1} = j_1^{-1}v_9j_1v_6^{-1} = 1, \\
j_1^{-1}v_7j_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= j_1^{-1}v_{11}j_1v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
t_1^{-1}a_1t_1d_1^{-1}c_1^{-1}v_6^{-1}v_9^{-1} &= t_1^{-1}b_1t_1d_1^{-1}a_1^{-1}v_6^{-1}v_9^{-1} = 1, \\
t_1^{-1}c_1t_1d_1^{-1}b_1^{-1}v_6^{-1}v_9^{-1} &= t_1^{-1}d_1t_1c_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_9^{-1} = 1, \\
g_1^2v_1^{-1}v_3^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= (g_1a_1)^3v_{10}^{-1}v_{11}^{-1} = 1, \\
(g_1b_1)^3v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1} &= (g_1c_1)^3 = 1, \quad (i_1j_1)^3 = 1 \\
(g_1t_1)^2v_1^{-1}v_3^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
h_1^2v_6^{-1} = 1, \quad h_1^{-1}b_1h_1d_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1} &= 1, \\
h_1^{-1}c_1h_1c_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1} &= h_1^{-1}d_1h_1d_1^{-1}v_6^{-1} = 1, \\
h_1^{-1}t_1h_1t_1v_9^{-1} &= (g_1h_1)^3v_3^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
i_1^{-1}a_1i_1d_1^{-1}c_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
i_1^{-1}b_1i_1d_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
i_1^{-1}c_1i_1d_1^{-1}c_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
i_1^{-1}d_1i_1d_1^{-1}c_1^{-1}b_1^{-1} &= i_1^{-1}t_1i_1t_1 = i_1^{-1}g_1i_1g_1^{-1}t_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
(h_1i_1)^3v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{10}^{-1} &= j_1^{-1}a_1j_1c_1^{-1}b_1^{-1}a_1^{-1} = 1, \\
j_1^{-1}d_1j_1d_1^{-1}c_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1} &= j_1^{-1}t_1j_1t_1 = 1, \quad j_1^{-1}h_1j_1h_1^{-1}t_1^{-1} = 1.
\end{aligned}$$

Proof. The 2 irreducible $F\mathcal{M}_{23}$ -modules V_i , $i = 1, 2$, occur as composition factors with multiplicity 1 in the permutation module $(1_{\mathcal{M}_{22}})^{\mathcal{M}_{23}}$ of degree 23 where $\mathcal{M}_{22} = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1 \rangle$. They are dual to each other. Using the faithful permutation representation of \mathcal{M}_{23} stated in (a) and the Meat-axe Algorithm implemented in MAGMA one obtains the generating matrices of \mathcal{M}_{23} stated in (b) defining V_1 . Their dual matrices define V_2 . They are stated in (c).

(d) The cohomological dimensions $\dim_F[H^2(\mathcal{M}_{23}, V_i)]$, $i = 1, 2$, have been calculated by means of MAGMA using Holt's Algorithm 7.4.5 of [12]. Its hypothesis is satisfied by the presentation of \mathcal{M}_{23} stated in Definition 8.2.1 of [12] and all the data of (a), (b) and (c). It follows that $\dim_F[H^2(\mathcal{M}_{23}, V_1)] = 1$ and $\dim_F[H^2(\mathcal{M}_{23}, V_2)] = 0$.

(e) This statement is checked easily with MAGMA.

(g) The presentation of the non-split extension E has been obtained by means of the commands `ExtensionProcess` and `Extension` of Holt's Algorithm 7.4.5 of [12] implemented in MAGMA [7]. This completes the proof.

□

The first statement of the following subsidiary lemma is mainly due to Paul Young.

Lemma 3.2. *With the notations of Lemma 3.1 the following statements hold:*

- (a) *The non-split extension E of \mathcal{M}_{23} by its simple module V_1 has a faithful permutation representation PE of degree 1012 with stabilizer U generated by the two elements $(i_1^{-1}j_1^{-1}b_1g_1)^2$ and $(i_1^{-1}b_1t_1h_1^{-1})^4$.*
- (b) $E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1 \rangle$.
- (c) *E has 3 conjugacy classes of 2-central involutions. They are represented by $z_1 = (d_1g_1)^5$, $z_2 = h_1^2$ and $z_3 = g_1^2$. Their centralizers have orders $|C_E(z_1)| = 2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$, $|C_E(z_2)| = 2^{18} \cdot 3^2 \cdot 5 \cdot 7$ and $|C_E(z_3)| = 2^{18} \cdot 3^2 \cdot 5$.*
- (d) $D = C_E(z_1) = \langle x_1, y_1 \rangle$ and $E = \langle D, e_1 \rangle$ where $x_1 = a_1$, $y_1 = b_1g_1h_1i_1$ and $e_1 = j_1$ have respective orders 2, 14 and 2.
- (e) $E = \langle x_1, y_1, e_1 \rangle$ has 56 conjugacy classes. A system of representatives is given in Table A.1.
- (f) $D = \langle x_1, y_1 \rangle$ has 69 conjugacy classes. A system of representatives is given in Table A.2.
- (g) *The character tables of E and D are given in Tables B.1 and B.2, respectively.*

Proof. (a) The two generators of the stabilizer U have been found by means of a program due to P. Young. Using the MAGMA command `CosetAction(E,U)` one obtains a faithful permutation representation PE of E with stabilizer U and degree 1012.

(b), (c) and (d) These statements are easily checked using the permutation representation PE and MAGMA.

(e) and (f) The faithful permutation representation PE of E , MAGMA and Kratzer's Algorithm 5.3.18 of [12] are employed to calculate a system of representatives of all conjugacy classes of D and E in terms of their generators given in (d). The results are stated in Tables A.1 and A.2.

The results of (g) were computed by means of MAGMA.

□

4. CONSTRUCTION OF THE 2-CENTRAL INVOLUTION CENTRALIZER OF Fi_{23}

Let z_1 denote the central involution of the extension E of \mathcal{M}_{23} defined in Lemma 3.1 and let $D = C_E(z_1)$. Then in the following subsidiary result it is shown that $D_1 = D/\langle z_1 \rangle$ is isomorphic to the extension $E_2 = E(\text{Fi}_{22})$ of Lemma 3.1. As proved in [9], the Fischer's simple group $G_1 \cong \text{Fi}_{22}$ can be constructed from the group D_1 by Algorithm 7.4.8 of [12]. Unfortunately, (in 2007) MAGMA was not able to perform all steps of Holt's Algorithm to establish the 2-fold cover H of G_1 from the presentation of the given group G_1 . Therefore the author constructs first all the central extensions H_2 of the centralizer $H_1 = C_{G_1}(t)$ of a 2-central involution t of G_1 by a cyclic group of order 2. It will be shown that one of them has a Sylow 2-subgroup which is isomorphic to the ones of D . Thus H_2 which will be a subgroup of H is uniquely determined up to isomorphism.

The group H , with center $Z(H) = \langle z_1 \rangle$ such that $H/Z(H) \cong \text{Fi}_{22}$, isomorphic to the centralizer of a 2-central involution of the target group \mathfrak{G} , is then constructed by means of Algorithm 2.1 as a matrix subgroup \mathfrak{H} of $\text{GL}_{352}(17)$. In this way a faithful permutation representation of degree 28160 and a presentation of H are built. All these results are described in this section.

Proposition 4.1. *Keep the notations of Lemma 3.1 and 3.2, and let $E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1, v_i \mid 1 \leq i \leq 11 \rangle$ be the nonsplit extension of \mathcal{M}_{23} by its simple module V_1 of dimension 11 over $F = \text{GF}(2)$, and $D = C_E(z_1)$, where $z_1 = (d_1 g_1)^5$. Then the following statements hold:*

- (a) $z_1 = (d_1 g_1)^5$ is a 2-central involution of E with centralizer $D = C_E(z_1)$ of order $2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$.
- (b) $D = \langle x_1, y_1 \rangle$, $z_1 = (x_1 y_1^2)^7$ and $E = \langle D, e_1 \rangle$, where $x_1 = a_1$, $y_1 = b_1 g_1 h_1 i_1$ and $e_1 = j_1$ have respective orders 2, 14 and 2. D has a center of order 2, generated by z_1 .
- (c) V is a unique normal subgroup of D of order 2^{11} . V is elementary abelian, and has a basis $\mathcal{B} = \{z_1, v_i \mid 1 \leq i \leq 10\}$, where

$$\begin{aligned} v_1 &= y_1^7, & v_2 &= (x_1 y_1 x_1)^7, & v_3 &= (x_1 y_1 x_1 y_1^2)^8, & v_4 &= (x_1 y_1^2 x_1 y_1)^8, \\ v_5 &= (y_1 x_1 y_1^2 x_1)^8, & v_6 &= (x_1 y_1 x_1 y_1^3)^6, & v_7 &= (x_1 y_1^3 x_1 y_1)^6, \\ v_8 &= (x_1 y_1^5)^7, & v_9 &= (y_1 x_1 y_1 x_1 y_1^2)^6, & v_{10} &= (y_1^2 x_1 y_1^3)^7. \end{aligned}$$

- (d) $V_1 = V/\langle z_1 \rangle$ has a complement W_1 in $D_1 = D/\langle z_1 \rangle$ such that $W_1 \cong \mathcal{M}_{22}$. In particular, $D_1 \cong E(\text{Fi}_{22})$ defined in Proposition 3.3 in [9] (in [9], $E(\text{Fi}_{22})$ appears as E_2).
- (e) Let $z_1 = (x_1 y_1^2)^7$ and let v_i be the 10 basis elements of \mathcal{B} given in (c). Then $D = \langle x_1, y_1 \rangle$ has the following set $\mathcal{R}(D)$ of defining relations:

$$\begin{aligned} z_1^2 &= 1, & (x_1, z_1) &= (y_1, z_1) = 1, \\ v_i^2 &= 1, & \text{for all } & 1 \leq i \leq 10, \\ (v_j, v_k) &= 1 & \text{for all } & 1 \leq j < k \leq 10, \\ x_1^2 &= y_1^{-7} v_1 = (x_1 y_1^{-1})^7 = (y_1^{-1} x_1 y_1 x_1)^5 = (y_1^{-2} x_1)^7 z_1 = 1, \\ (y_1^{-2} x_1 y_1^2 x_1 y_1^{-1} x_1)^3 v_2 &= (x_1 y_1^{-2} x_1 y_1)^5 v_1 v_2 v_3 v_4 v_7 v_8 v_{10} = 1, \\ (y_1 x_1 y_1 x_1 y_1^2 x_1 y_1 x_1 y_1^{-1} x_1 y_1^2)^2 v_1 v_3 v_5 v_6 v_9 z_1 &= 1, \\ x_1 y_1^{-1} x_1 y_1 x_1 y_1^2 x_1 y_1^{-1} x_1 y_1^{-1} x_1 y_1^2 x_1 y_1 x_1 y_1^{-1} x_1 y_1^2 x_1 y_1^2 x_1 y_1 v_1 v_3 v_6 v_8 z_1, \\ y_1 x_1 y_1^3 x_1 y_1 x_1 y_1^{-2} x_1 y_1^{-3} x_1 y_1 x_1 y_1^3 x_1 y_1^{-1} x_1 y_1^{-3} x_1 v_4 v_8 v_{10} &= 1, \end{aligned}$$

$$\begin{aligned}
 x_1 v_1 x_1^{-1} v_2 &= x_1 v_2 x_1^{-1} v_1 = x_1 v_3 x_1^{-1} v_5 = x_1 v_4 x_1^{-1} v_3 v_4 v_5 = x_1 v_5 x_1^{-1} v_3 = 1, \\
 x_1 v_6 x_1^{-1} v_1 v_2 v_6 &= x_1 v_7 x_1^{-1} v_1 v_2 v_7 = x_1 v_8 x_1^{-1} v_4 v_6 v_9 = 1, \\
 x_1 v_9 x_1^{-1} v_1 v_2 v_3 v_4 v_5 v_6 v_8 &= 1, \\
 x_1 v_{10} x_1^{-1} v_1 v_3 v_8 v_9 v_{10} z_1 &= y_1 v_1 y^{-1} v_1 = y_1 v_2 y^{-1} v_3 v_4 v_5 v_{10} = 1, \\
 y_1 v_3 y^{-1} v_1 v_3 v_4 z_1 &= y_1 v_4 y^{-1} v_5 = y_1 v_5 y^{-1} v_2 v_6 v_8 v_9 z_1 = y_1 v_6 y^{-1} v_9 = 1, \\
 y_1 v_7 y^{-1} v_1 v_2 v_6 &= y_1 v_8 y^{-1} v_1 v_2 v_3 v_4 z_1 = y_1 v_9 y^{-1} v_1 v_6 v_7 v_9 z_1 = 1, \\
 y_1 v_{10} y^{-1} v_2 v_4 v_5 v_7 z_1 &= 1.
 \end{aligned}$$

(f) The involution $t = (x_1 y_1 x_1 y_1^3)^6$ of D has a centralizer $T = C_D(t)$ of order $2^{18} \cdot 3 \cdot 5$.

Proof. (a) and (b) are restatements of Lemma 3.2.

(c) It can be checked by means of the MAGMA command

`Subgroups(D : A1 := 'Normal')`

that D has a unique normal subgroup V of order 2^{11} , and that V is elementary abelian. Now, by means of

`GetShortGens(sub<D|x_1,y_1>, V : exclude:=sub<D|z_1>)`

we get the 10 generators v_i which together with z_1 generate V . Since V is elementary abelian, we can regard the generators as a basis.

(d) As $z_1 \in V$ it follows that $V_1 = V/\langle z_1 \rangle$ is the unique normal subgroup of $D_1 = D/\langle z_1 \rangle$ of order 2^{10} . Clearly, V_1 is elementary abelian.

Applying the MAGMA command `CompositionFactors(D)` one sees that $D_1/V_1 \cong D/V \cong \mathcal{M}_{22}$. Thus Lemma 2.4(d) of [9] asserts that D_1 splits over V_1 .

Let Mx and My be the matrices of the generators x_1 and y_1 of D w.r.t. the basis \mathcal{B} of V . Then

$$Mx = \begin{pmatrix} 10000000000 \\ 00100000000 \\ 01000000000 \\ 00000100000 \\ 00011000000 \\ 00010000000 \\ 01100010000 \\ 01100001000 \\ 00001010010 \\ 01111110100 \\ 11010000111 \end{pmatrix}, \quad My = \begin{pmatrix} 10000000000 \\ 01000000000 \\ 00010000100 \\ 11011011011 \\ 00001011011 \\ 00001000000 \\ 01010001100 \\ 10010011110 \\ 11000111000 \\ 00000010000 \\ 11111000000 \end{pmatrix},$$

Both matrices are blocked lower triangular matrices with upper left diagonal blocks equal to 1 and lower diagonal 10×10 blocks Mx_1 and My_1 in $\text{GL}_{10}(2)$. Hence $D_1/V_1 \cong W_1$, where $W_1 = \langle Mx_1, My_1 \rangle \leq \text{GL}_{10}(2)$. Applying the MAGMA command `FPGROUP(sub<GL(10,2)|Mx1,My1>)` to W_1 one obtains the following set $\mathcal{R}(W_1)$ of defining relations of $W_1 = \langle x_1, y_1 \rangle$:

$$\begin{aligned}
 x_1^2 &= 1, & y_1^7 &= 1, \\
 (x_1 y_1^{-1})^7 &= 1, & (y_1^{-1} x_1 y_1 x_1)^5 &= 1, & (y_1^{-2} x_1)^7 &= 1, \\
 (y_1^{-2} x_1 y_1^2 x_1 y_1^{-1} x_1) &= 1, & (x_1 y_1^{-2} x_1 y_1)^5 &= 1, \\
 (y_1 x_1 y_1 x_1 y_1^2 x_1 y_1 x_1 y_1^{-1} x_1 y_1^2)^2 &= 1, \\
 x_1 y_1^{-1} x_1 y_1 x_1 y_1^2 x_1 y_1^{-1} x_1 y_1^{-1} x_1 y_1^2 x_1 y_1 x_1 y_1^{-1} x_1 y_1^2 x_1 y_1^2 x_1 y_1 &= 1, \\
 y_1 x_1 y_1^3 x_1 y_1 x_1 y_1^{-2} x_1 y_1^{-3} x_1 y_1 x_1 y_1^3 x_1 y_1^{-1} x_1 y_1^{-3} x_1 &= 1.
 \end{aligned}$$

Let PD_1 be the faithful permutation representation of D_1 of degree 1024 with stabilizer W_1 . Let $E_2 = E(\text{Fi}_{22})$ be the finitely presented group of Lemma 2.4(f)

of [9] with faithful permutation representation PE_2 defined in Lemma 2.6(a) of [9]. By means of the MAGMA command `IsIsomorphic(PE_2,PD_1)` it is verified that D_1 is isomorphic to E_2 .

(e) By means of MAGMA the semidirect product $D_1 = \langle x_1, y_1, v_i \mid 1 \leq i \leq 10 \rangle$ of W_1 by $V_1 = \langle v_i \mid 1 \leq i \leq 10 \rangle$ has a set of defining relations $\mathcal{R}(D_1)$ consisting of $\mathcal{R}(W_1)$ and the following set of relations:

$$\begin{aligned}
v_i^2 &= 1, (v_j, v_k) = 1 \quad \text{for all } 1 \leq i, j, k \leq 10, \\
x_1 v_1 x_1^{-1} v_2 &= x_1 v_2 x_1^{-1} v_1 = x_1 v_3 x_1^{-1} v_5 = 1, \\
x_1 v_4 x_1^{-1} v_3 v_4 v_5 &= x_1 v_5 x_1^{-1} v_3 = 1, \\
x_1 v_6 x_1^{-1} v_1 v_2 v_6, x_1 v_7 x_1^{-1} v_1 v_2 v_7 &= 1, \\
x_1 v_8 x_1^{-1} v_4 v_6 v_9, x_1 v_9 x_1^{-1} v_1 v_2 v_3 v_4 v_5 v_6 v_8 &= 1, \\
x_1 v_{10} x_1^{-1} v_1 v_3 v_8 v_9 v_{10} &= y_1 v_1 y_1^{-1} v_1 = 1, \\
y_1 v_2 y_1^{-1} v_3 v_4 v_5 v_{10} &= y_1 v_3 y_1^{-1} v_1 v_3 v_4 = 1, \\
y_1 v_4 y_1^{-1} v_5 &= y_1 v_5 y_1^{-1} v_2 v_6 v_8 v_9 = 1, \\
y_1 v_6 y_1^{-1} v_9 &= y_1 v_7 y_1^{-1} v_1 v_2 v_6 = 1, \\
y_1 v_8 y_1^{-1} v_1 v_2 v_3 v_4 &= y_1 v_9 y_1^{-1} v_1 v_6 v_7 v_9 = y_1 v_{10} y_1^{-1} v_2 v_4 v_5 v_7 = 1.
\end{aligned}$$

By (c) each v_i is a word in the generators x_1 and y_1 of D . Evaluating the relations of $\mathcal{R}(D_1)$ in D , we get that each relation has value equal to either 1 or z_1 . Appending z_1 to appropriate relations, we get the defining set of relations $\mathcal{R}(D)$, as written in the statement. \square

Proposition 4.2. *Keep the notations of Proposition 4.1. The following statements hold:*

- (a) *Let $H_1 = \langle r_i \mid 1 \leq i \leq 14 \rangle \cong H(\text{Fi}_{22})$ be the finitely presented group constructed in Proposition 3.3(n) of [9] (for the notation, h_i 's in Proposition 3.3(n) of [9] are replaced by r_i 's). Then H_1 has a central extension $H_2 = \langle k_j \mid 1 \leq j \leq 15 \rangle$ of order $2^{18} \cdot 3^4 \cdot 5$ having the following set of $\mathcal{R}(H_2)$ of defining relations:*

$$\begin{aligned}
k_1^2 &= k_2^5 = k_3^3 = k_4^3 = k_5^2 = k_6^2 = k_7^2 = 1, \\
k_8^2 &= k_9^2 = k_{10}^4 = k_{11}^2 = k_{12}^2 = k_{13}^2 = k_{14}^2 = k_{15}^2 = 1, \\
(k_i, k_{15}) &= 1 \quad \text{for } 1 \leq i \leq 14, \\
(k_i, k_{14}) &= 1 \quad \text{for } 1 \leq i \leq 13, \\
(k_i, k_{13}) &= 1 \quad \text{for } 2 \leq i \leq 12, \\
k_1^{-1} k_{13} k_1 k_{13}^{-1} k_{14}^{-1} &= 1, \\
(k_3, k_4) &= 1, \quad k_3^{-1} k_2 k_1 k_2^{-1} k_3 k_1 = 1, \\
(k_2^{-1} k_1)^4 &= 1, \quad k_3^{-1} k_2 k_3^{-1} k_1 k_2 k_3 k_1 k_2^{-1} = 1, \\
(k_2 k_3^{-1})^4 &= 1, \quad (k_4, k_2^{-1}, k_4) = 1, \\
k_1 k_4^{-1} k_1 k_4^{-1} k_1 k_4 k_1 k_4 &= 1, \quad k_2^{-1} k_3^{-1} k_1 k_2^{-2} k_3 k_4 k_1 k_4^{-1} = 1, \\
k_2 k_3^{-1} k_2 k_4 k_2^2 k_3^{-1} k_2^{-1} k_3 k_4 &= 1, \quad k_1 k_5 k_1^{-1} k_5 k_6 k_{13}^{-1} = 1, \\
k_1 k_6 k_1^{-1} k_6 k_{14}^{-1} &= k_1 k_7 k_1^{-1} k_7 k_8 k_{14}^{-1} = k_1 k_8 k_1^{-1} k_8 = 1,
\end{aligned}$$

$$\begin{aligned}
 &k_1 k_9 k_1^{-1} k_{11} k_{13}^{-1} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_1 k_{10} k_1^{-1} k_6 k_9 k_{10} k_{11} k_{12} k_{13}^{-1} k_{15}^{-1} = 1, \\
 &k_1 k_{11} k_1^{-1} k_9 k_{13}^{-1} k_{15}^{-1} = k_1 k_{12} k_1^{-1} k_{12} k_{14}^{-1} = 1, \\
 &k_2 k_5 k_2^{-1} k_5 k_6 k_7 k_{14}^{-1} k_{15}^{-1} = k_2 k_6 k_2^{-1} k_7 k_8 k_{13}^{-1} k_{14}^{-1} = 1, \\
 &k_2 k_7 k_2^{-1} k_6 k_7 k_{14}^{-1} k_{15}^{-1} = k_2 k_8 k_2^{-1} k_5 k_6 k_7 k_8 = 1, \\
 &k_2 k_9 k_2^{-1} k_5 k_6 k_{10} k_{11} k_{13}^{-1} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_2 k_{10} k_2^{-1} k_7 k_8 k_9 k_{13}^{-1} = k_2 k_{11} k_2^{-1} k_5 k_{12} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_2 k_{12} k_2^{-1} k_6 k_7 k_8 k_{10} k_{12} = k_3 k_5 k_3^{-1} k_6 k_{14}^{-1} = 1, \\
 &k_3 k_6 k_3^{-1} k_5 k_6 k_{13}^{-1} = k_3 k_7 k_3^{-1} k_5 k_8 k_{15}^{-1} = 1, \\
 &k_3 k_8 k_3^{-1} k_5 k_6 k_7 k_8 = k_3 k_9 k_3^{-1} k_5 k_9 k_{10} k_{14}^{-1} = 1, \\
 &k_3 k_{10} k_3^{-1} k_6 k_9 k_{15}^{-1} = k_3 k_{11} k_3^{-1} k_9 k_{12} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_3 k_{12} k_3^{-1} k_5 k_{10} k_{11} k_{12} k_{13}^{-1} k_{14}^{-1} = 1, \\
 &k_4 k_5 k_4^{-1} k_5 k_6 k_{13}^{-1} k_{14}^{-1} = 1, \\
 &k_4 k_6 k_4^{-1} k_5 k_{14}^{-1} = k_4 k_7 k_4^{-1} k_6 k_{10} k_{11} k_{12} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_4 k_8 k_4^{-1} k_5 k_6 k_{10} k_{11} k_{13}^{-1} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_4 k_9 k_4^{-1} k_5 k_6 k_{11} k_{12} k_{14}^{-1} = 1, \\
 &k_4 k_{10} k_4^{-1} k_9 k_{10} k_{12} k_{13}^{-1} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_4 k_{11} k_4^{-1} k_5 k_6 k_7 k_8 k_9 k_{11} k_{13}^{-1} k_{15}^{-1} = 1, \\
 &k_4 k_{12} k_4^{-1} k_5 k_7 k_{10} k_{11} k_{14}^{-1} = k_{10}^2 k_{14}^{-1} k_{15}^{-1} = 1, \\
 &(k_5, k_6) = (k_5, k_7) = (k_5, k_8) = (k_5, k_9) = 1, \\
 &k_5^{-1} k_{10}^{-1} k_5 k_{10} k_{14}^{-1} k_{15}^{-1} = k_5^{-1} k_{11}^{-1} k_5 k_{11} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &(k_5, k_{12}) = (k_6, k_7) = (k_6, k_8) = 1, \\
 &k_6^{-1} k_9^{-1} k_6 k_9 k_{14}^{-1} k_{15}^{-1} = k_6^{-1} k_{10}^{-1} k_6 k_{10} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_6^{-1} k_{11}^{-1} k_6 k_{11} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &(k_6, k_{12}) = (k_7, k_8) = (k_7, k_9) = (k_8, k_{10}) = (k_8, k_{12}) = 1, \\
 &k_7^{-1} k_{10}^{-1} k_7 k_{10} k_{14}^{-1} k_{15}^{-1} = k_7^{-1} k_{11}^{-1} k_7 k_{11} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_7^{-1} k_{12}^{-1} k_7 k_{12} k_{14}^{-1} k_{15}^{-1} = k_8^{-1} k_9^{-1} k_8 k_9 k_{14}^{-1} k_{15}^{-1} = 1, \\
 &k_8^{-1} k_{11}^{-1} k_8 k_{11} k_{14}^{-1} k_{15}^{-1} = 1, \\
 &(k_9, k_{10}) = (k_9, k_{11}) = (k_9, k_{12}) = (k_{10}, k_{12}) = (k_{11}, k_{12}) = 1, \\
 &k_{10}^{-1} k_{11}^{-1} k_{10} k_{11} k_{14}^{-1} k_{15}^{-1} = 1.
 \end{aligned}$$

- (b) H_2 has a faithful permutation representation of degree 2048 with stabilizer $U_2 = \langle k_1, k_2, k_3, k_4 \rangle$.
- (c) H_2 has a Sylow 2-subgroup S_2 generated by the four involutions $m_1 = k_1$, $m_2 = (k_2 k_1 k_2 k_4 k_5)^6$, $m_3 = (k_2^3 k_1 k_2)^2$ and $m_4 = (k_4^2 k_2 k_4 k_2 k_4)^6$.
- (d) A_2 is the unique maximal elementary abelian normal subgroup of S_2 of order 2^{11} , and is generated by the following 11 elements:

$$\begin{aligned}
 &m_4, (m_2 m_4)^2, (m_3 m_4)^2, (m_1 m_3 m_4)^4, (m_2 m_3 m_4)^4, \\
 &(m_1 m_2 m_3 m_4)^8, (m_1 m_3 m_1 m_4)^2, (m_2 m_3 m_2 m_4)^2, \\
 &(m_2 m_3 m_4 m_2)^2, (m_1 m_2 m_3 m_1 m_4)^4, \text{ and } (m_1 m_2 m_3 m_2 m_4)^4.
 \end{aligned}$$

- (e) $D_2 = N_{H_2}(A_2) = \langle p_2, q_2 \rangle$ where $p_2 = k_1 k_2$ and $q_2 = k_1 (k_1 k_5 k_4 k_2)^5$. Here p_2 and q_2 have orders 4 and 6, respectively.

- (f) $H_2 = \langle D_2, h_2 \rangle = \langle p_2, q_2, h \rangle$ where $h_2 = k_4$ has order 3.
 (g) There is an isomorphism $\alpha : D_2 \rightarrow T$ s.t. $\alpha(p_2) = p_1 = (u_1 u_2 u_4 u_6 u_7)^3$ and $\alpha(q_2) = q_1 = (w_1 w_2 w_3 w_4 w_2 w_3 w_4)^5$, where

$$\begin{aligned} u_1 &= (x_1 y_1^2)^7, & u_2 &= (x_1 y_1 x_1 y_1^3)^6, & u_3 &= (y_1^2 x_1 y_1^3)^7, \\ u_4 &= (y_1^3 x_1 y_1^2)^7, & u_5 &= x_1 y_1 x_1 y_1 x_1 y_1 x_1 y_1^2 x_1 y_1 x_1 y_1, \\ u_6 &= (y_1^3 x_1 y_1^2 x_1 y_1^5 x_1 y_1^3 x_1 y_1)^2, & u_7 &= (y_1^5 x_1 y_1^2 x_1 y_1 x_1 y_1 x_1 y_1 x_1 y_1^2 x_1)^4. \end{aligned}$$

and

$$\begin{aligned} w_1 &= (s_1)^7, & w_2 &= (s_1^4 s_2^2 s_1^2 s_2^2 s_1)^2, \\ w_3 &= s_1^2 s_2 s_1^3 s_2^2 s_1^3, & w_4 &= (s_1^4 s_2^2 s_1^2 s_2 s_1^3 s_2 s_1^3)^2, \end{aligned}$$

where $s_1 = x_1 y_1 x_1$ and $s_2 = (x_1 y_1 x_1 y_1^3)^2$.

- (h) So, $D = \langle p_1, q_1, x_1, y_1 \rangle$, $T = C_D(t) = \langle p_1, q_1 \rangle$, $H_2 = \langle p_2, q_2, h_2 \rangle$, and $D_2 = N_{H_2}(A) = \langle p_2, q_2 \rangle$.
 (i) A system of representatives r_i of the 189 conjugacy classes of H_2 and the corresponding centralizers orders $|C_{H_2}(r_i)|$ are given in Table A.3.
 (j) A system of representatives d_i of the 151 conjugacy classes of D_2 and the corresponding centralizers orders $|C_{D_2}(d_i)|$ are given in Table A.4.
 (k) The character tables of H_2 and D_2 are given in Tables B.4 and B.5, respectively.

Proof. (a) By Table A.8 of [9] the simple group Fi_{22} has a unique conjugacy class of 2-central involutions, represented by an element u . Theorem 5.1 of [9] asserts that $H_1 = C_{\text{Fi}_{22}}(u)$ is isomorphic to the finitely presented group $H_1 = \langle r_i \mid 1 \leq i \leq 14 \rangle$ constructed in Proposition 3.3(n) of [9] (with notation changed from h_i in [9] to r_i here). H_1 has a faithful permutation representation PH_1 of degree 1024 with stabilizer $\langle r_1, r_2, r_3, r_4 \rangle$ by Lemma 3.4 of [9].

Let $F = \text{GF}(2)$, and let M be the trivial FH_1 -module, which can be constructed in MAGMA by the commands

```
FEalg := MatrixAlgebra< FiniteField(2), 1 | [[1] : i in [1..14]]>
```

and

```
M := GModule(PH_1, FEalg).
```

Using the faithful permutation representation PH_1 and the MAGMA command

```
CohomologicalDimension(PH_1, M, 2)
```

we get that this cohomological dimension is 3. For each of the eight 2-cocycles (a, b, c) with $a, b, c \in F$, MAGMA constructs a finitely presented group $E_{(a,b,c)}$ by means of its commands

```
P := ExtensionProcess(PH_1, M, H_1),
E_{a,b,c} := Extension(P, [a,b,c]).
```

Since D is a non-split extension of D_1 by $\langle z_1 \rangle$ the split extension $E_{(0,0,0)}$ can be neglected.

It is checked by means of MAGMA using the faithful permutation representation PD of D of degree 1012 that $E_{(1,1,0)}$ is a non-split extension of order $2^{18} \cdot 3^4 \cdot 5$ of H_1 having a Sylow 2-subgroup which is isomorphic to any of the Sylow 2-subgroups of D . The presentation of $H_2 = E_{(1,1,0)}$ is given in the statement.

(b) The MAGMA command `CosetAction(H_2, U_2)` for the subgroup $U_2 = \langle k_1, k_2, k_3, k_4 \rangle$ of the finitely presented group H_2 gives the faithful permutation

representation of H_2 of degree 2048. We know it is faithful because the order of the resulting permutation group has the correct order $2^{18} \cdot 3^4 \cdot 5$.

(c) A Sylow 2-subgroup S_2 of H_2 can be obtained by the MAGMA command

$$\text{S}_2 := \text{SylowSubgroup}(\text{H}_2, 2)$$

and its generators can be obtained by the command

$$\text{GetShortGens}(\text{H}_2, \text{S}_2),$$

which gave the four generators $m_1 = k_1$, $m_2 = (k_2 k_1 k_2 k_4 k_5)^6$, $m_3 = (k_2^3 k_1 k_2)^2$ and $m_4 = (k_4^2 k_2 k_4 k_2 k_4)^6$, as asserted. The four generators of S_2 all have order 2.

(d) It can be verified by the MAGMA command

$$\text{Subgroups}(\text{S}_2 : \text{Al} := \text{'Normal'}, \text{IsElementaryAbelian} := \text{true})$$

that S_2 has a unique maximal elementary abelian subgroup A_2 of order 2^{11} . Now the generators for A_2 can be obtained by the command

$$\text{GetShortGens}(\text{sub}\langle \text{H}_2 | m_1, m_2, m_3, m_4 \rangle, \text{A}_2),$$

which gave the 11 generators as written in the statement.

(e) Letting

$$\text{D}_2 := \text{Normalizer}(\text{H}_2, \text{A}_2)$$

yields a subgroup D_2 of H_2 of order $2^{18} \cdot 3 \cdot 5$, and its generators can be obtained by the command

$$\text{GetShortGens}(\text{H}_2, \text{D}_2),$$

which gave the two generators $p_2 = k_1 k_2$ and $q_2 = k_1 (k_1 k_5 k_4 k_2)^5$, having orders 4 and 6, respectively.

(f) It can be verified with MAGMA that $H_2 = \langle D_2, k_4 \rangle = \langle p_2, q_2, k_4 \rangle$. We let $h_2 = k_4$, and it is easy to check with MAGMA that h_2 has order 3.

(g) The isomorphism $\alpha : D_2 \rightarrow T$ can be obtained by the MAGMA command

$$\text{IsIsomorphic}(\text{D}_2, \text{T}).$$

Recall that T is a subgroup of $D = \langle x_1, y_1 \rangle$. For documentation of this isomorphism α we need to get words for $\alpha(p_2)$ and $\alpha(q_2)$ in terms of x_1 and y_1 . Now, for convenience, let $p_1 = \alpha(p_2)$ and $q_1 = \alpha(q_2)$.

In order to obtain short words for p_1 and q_1 , the author employed Strategy 1.22. First, it is checked with MAGMA that $N_D(\langle p_1 \rangle)$ has order 2^8 , and it is obtained by `GetShortGens` that $N_D(\langle p_1 \rangle) = \langle u_1, u_2, u_3, u_4 u_5, u_6, u_7 \rangle$, where the words u_1 through u_7 are as written in the statement. Then, the command `LookupWord(sub<D|u_1,u_2,u_3,u_4,u_5,u_6,u_7>, p_1)` gave the answer $p_1 = (u_1 u_2 u_4 u_6 u_7)^3$.

Note that q_1 has order 6, since so does q_2 . It can be checked with MAGMA that the $N_D(\langle q_1 \rangle)$ has order $2^6 \cdot 3^2$ and that $C_D(q_1^3)$ has order $2^{17} \cdot 3^2 \cdot 5 \cdot 7$. It is easy to observe that $N_D(\langle q_1 \rangle) \leq C_D(q_1^3)$. Employing Strategy 1.21, we can find out that $C_D(q_1^3) = \langle s_1, s_2 \rangle$ and $N_D(\langle q_1 \rangle) = \langle w_1, w_2, w_3, w_4 \rangle$, where $s_1 = x_1 y_1 x_1$, $s_2 = (x_1 y_1 x_1 y_1^3)^2$, $w_1 = (s_1)^7$, $w_2 = (s_1^4 s_2^2 s_1^2 s_2^2 s_1)^2$, $w_3 = s_1^2 s_2 s_1^3 s_2^2 s_1^3$, and $w_4 = (s_1^4 s_2^2 s_1^2 s_2 s_1^3 s_2^3 s_1)^2$. Finally, the command `LookupWord(sub<D|w_1,w_2,w_3,w_4>, q_1)` successfully gave the word for q_1 , namely $q_1 = (w_1 w_2 w_3 w_4 w_2 w_3 w_4)^5$.

(h) From (g) we know that $D_2 = \langle p_2, q_2 \rangle$ and $\alpha : D_2 \rightarrow T$ is an isomorphism, and also that $p_1 = \alpha(p_2)$ and $q_1 = \alpha(q_2)$. Therefore we have $T = \langle p_1, q_1 \rangle$. Since $D = \langle x_1, y_1 \rangle$, it is clear that $D = \langle p_1, q_1, x_1, y_1 \rangle$. It is verified in (f) that $H_2 = \langle D_2, k_4 \rangle = \langle p_2, q_2, k_4 \rangle$.

The results of (i) and (j) can be obtained by applying Kratzer's Algorithm 5.3.18 of [12] in MAGMA to the relevant groups, with generators as stated in (h). The results of (k) were computed by means of MAGMA.

□

It will be shown in the rest of this section that the free product $H_2 *_{D_2} D$ of H_2 and D with amalgamated subgroup D_2 has an irreducible 352-dimensional faithful representation over $\text{GF}(17)$ which gives rise to the group H such that $|Z(H)| = 2$ and $H/Z(H) \cong \text{Fi}_{22}$.

Theorem 4.3. *Keep the notations in Proposition 4.2. Let $K = \text{GF}(17)$. Using the notations of the character tables of B.4, B.5, and B.2 of $H_2 = \langle p_2, q_2, h_2 \rangle$, $D_2 = \langle p_2, q_2 \rangle$, $D = \langle p_1, q_1, x_1, y_1 \rangle$, the following statements hold:*

- (a) *The smallest degree of a non-trivial compatible pair $(\chi, \tau) \in \text{mfchar}_{\mathbb{C}}(H_2) \times \text{mfchar}_{\mathbb{C}}(D)$ is 352.*
- (b) *There are exactly two compatible pairs $(\chi, \tau), (\chi', \tau') \in \text{mfchar}_{\mathbb{C}}(H_2) \times \text{mfchar}_{\mathbb{C}}(D)$ of degree 352 of $H_2 = \langle D_2, h \rangle$ and $D = \langle \alpha(D_2), x, y \rangle$:*

$$(\chi, \tau) = (\chi_{57} + \chi_{40} + \chi_{41}, \tau_9 + \tau_{10})$$

and

$$(\chi', \tau') = (\chi_{31} + \chi_{34} + \chi_{40} + \chi_{41}, \tau_9 + \tau_{10})$$

with common restriction

$$\chi|_{D_2} = \tau|_{\alpha(D_2)} = \chi'|_{D_2} = \tau'|_{\alpha(D_2)} = \psi_{26} + \psi_{27} + \psi_{59} + \psi_{63} + \psi_{70} + \psi_{73},$$

where irreducible characters with bold face indices denote faithful irreducible characters.

- (c) *Let \mathfrak{V} and \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple multiplicity-free 352-dimensional modules of H_2 and D over $F = \text{GF}(17)$ corresponding to the compatible pair χ, τ , respectively.*

Let $\kappa_{\mathfrak{V}} : H_2 \rightarrow \text{GL}_{352}(17)$ and $\kappa_{\mathfrak{W}} : D \rightarrow \text{GL}_{352}(17)$ be the representations of H_2 and D afforded by the modules \mathfrak{V} and \mathfrak{W} , respectively.

Let $\mathfrak{p}_2 = \kappa_{\mathfrak{V}}(p_2)$, $\mathfrak{q}_2 = \kappa_{\mathfrak{W}}(q_2)$, $\mathfrak{h}_2 = \kappa_{\mathfrak{W}}(h_2)$ in $\kappa_{\mathfrak{V}}(H_2) \leq \text{GL}_{352}(17)$. Then the following assertions hold:

- (1) *$\mathfrak{V}|_{D_2} \cong \mathfrak{W}|_{\alpha(D_2)}$, and there is a transformation matrix $\mathcal{T}_1 \in \text{GL}_{352}(17)$ such that*

$$\mathfrak{p}_2 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(p_1) \mathcal{T}_1, \quad \mathfrak{q}_2 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(q_1) \mathcal{T}_1.$$

- (2) *There is a transformation matrix $\mathcal{S}_1 \in \text{GL}_{352}(17)$ such that*

$$\mathfrak{p}_2 = \mathcal{S}_1^{-1} \mathfrak{p}_2 \mathcal{S}_1, \quad \mathfrak{q}_2 = \mathcal{S}_1^{-1} \mathfrak{q}_2 \mathcal{S}_1,$$

and that if we let

$$\mathfrak{r}_2 = (\mathcal{T}_1 \mathcal{S}_1)^{-1} \kappa_{\mathfrak{W}}(x_1) \mathcal{T}_1 \mathcal{S}_1 \in \text{GL}_{352}(17), \quad \text{and}$$

$$\mathfrak{h}_2 = (\mathcal{T}_1 \mathcal{S}_1)^{-1} \kappa_{\mathfrak{W}}(y_1) \mathcal{T}_1 \mathcal{S}_1 \in \text{GL}_{352}(17),$$

then $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2, \mathfrak{h}_2 \rangle = \langle \mathfrak{r}_2, \mathfrak{h}_2 \rangle$ satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of [12]. The proof showing that \mathfrak{H} satisfies the Sylow 2-subgroup test is split into two parts; first half is shown in the proof of this theorem (namely, the order of $\mathfrak{h}_2 \mathfrak{r}_2$ has to be the order of an element in 2Fi_{22} ; it turned out to be 12 in this case), and the other half in Theorem 4.5.

- (3) *The three generating matrices of \mathfrak{H} are stated in [10].*

- (d) *The construction shown above in (c) can be applied to the compatible pair χ', τ' . However, there is no solution in this case which satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of [12].*

Proof. (a) The character tables of the groups H_2, D_2 , and D are stated in the appendix. In the following we use their notations. Using MAGMA and the character tables of H_2, D_2 , and D and the fusion of the classes of D_2 in H_2 and $T = C_D(t) (\cong D_2)$ in D , an application of Kratzer's Algorithm 7.3.10 of [12] yields the compatible pair stated in assertion (a).

(b) The application of Kratzer's Algorithm 7.3.10 of [12] also shows that the pairs (χ, τ) and (χ', τ') of (b) are all the compatibles pairs of degree 352 with respect to the fusion of the D_2 -classes into the H_2 - and into the D -classes.

(c) In order to construct the faithful irreducible representation \mathfrak{V} corresponding to the character $\chi = \chi_{57} + \chi_{40} + \chi_{41}$ of degree 352, the MAGMA command `LowIndexSubgroups(PH_2, 1500)` is applied, using the faithful permutation presentation PH_2 of H_2 of degree 2048. MAGMA found subgroups U_1, U_2 , and U_3 such that the followings hold.

U_1 is of index 480 in H_2 , and χ_{57} (dimension 160) is a constituent of the permutation character $(1_{U_1})^{H_2}$. The program `GetShortGens(PH_2, U_1)` gives a generating set of U_1 , so we have $U_1 = \langle p_2 h_2 p_2, (p_2 q_2 h_2 q_2)^3, (q_2 h_2^2 p_2)^4, (h_2 p_2 q_2 h_2 q_2)^6 \rangle$. Using Meat-axe Algorithm to the permutation module $(1_{U_1})^{H_2}$, the author obtained the irreducible KH_2 -module \mathfrak{V}_{57} over K corresponding to χ_{57} . Here, the permutation module $(1_{U_1})^{H_2}$ is obtained by applying the MAGMA command `PermutationMatrix` to the generators of the permutation group obtained by `CosetAction(PH_2, U_1)`.

U_2 is of index 512 in H_2 , and χ_{40} (dimension 96) is a constituent of the permutation character $(1_{U_2})^{H_2}$. By `GetShortGens` we get $U_2 = \langle (q_2 p_2 h_2)^5, (q_2 h_2^2)^4, (p_2 h_2 q_2 p_2)^4, (p_2^2 q_2^2 h_2)^5 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{U_2})^{H_2}$, the irreducible KH_2 -module \mathfrak{V}_{40} corresponding to χ_{40} is obtained.

U_3 is of index 512 in H_2 , and χ_{41} (dimension 96) is a constituent of the permutation character $(1_{U_3})^{H_2}$. By `GetShortGens` we get $U_3 = \langle q_2^3, (h_2 q_2)^3, (q_2 p_2 h_2^2)^6, (q_2 p_2 h_2 q_2^2 h_2)^5 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{U_3})^{H_2}$, the irreducible KH_2 -module \mathfrak{V}_{41} corresponding to χ_{41} is obtained.

In order to construct the faithful irreducible representation \mathfrak{W} corresponding to the character $\tau = \tau_9 + \tau_{10}$ of degree 352, the MAGMA command `LowIndexSubgroups(PD, 1500)` is applied, using the faithful permutation presentation PD of D of degree 1012. MAGMA found subgroups S_1 and S_2 such that the followings hold.

S_1 is of index 352 in D , and τ_9 (dimension 176) is a constituent of the permutation character $(1_{S_1})^D$. By `GetShortGens` we get $S_1 = \langle (q_1 p_1^3)^3, (x_1 q_1 p_1 q_1 x_1)^3, (x_1 q_1 x_1 p_1 q_1)^2 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_1})^D$, the irreducible KD -module \mathfrak{W}_9 corresponding to τ_9 is obtained.

S_2 is of index 352 in D , and τ_{10} (dimension 176) is a constituent of the permutation character $(1_{S_2})^D$. By `GetShortGens` we get $S_2 = \langle (p_1 x_1 q_1)^4, x_1 q_1 x_1, (p_1 q_1 x_1 p_1)^2 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_2})^D$, the irreducible KD -module \mathfrak{W}_{10} corresponding to τ_{10} is obtained.

Thus, we can obtain \mathfrak{V} and \mathfrak{W} by letting $\mathfrak{V} = \mathfrak{V}_{57} \oplus \mathfrak{V}_{40} \oplus \mathfrak{V}_{41}$ and $\mathfrak{W} = \mathfrak{W}_9 \oplus \mathfrak{W}_{10}$, corresponding to the characters $\chi = \chi_{57} + \chi_{40} + \chi_{41}$ and $\tau = \tau_9 + \tau_{10}$, respectively. Direct summation of modules here means block diagonal joining of

matrices, in matrix sense. For example, $\mathfrak{p}_2 = \kappa_{\mathfrak{V}}(p_2)$ is obtained by

$$\kappa_{\mathfrak{V}}(p_2) = \text{diag}(\kappa_{\mathfrak{V}_{57}}(p_2), \kappa_{\mathfrak{V}_{40}}(p_2), \kappa_{\mathfrak{V}_{41}}(p_2)),$$

where $\kappa_{\mathfrak{V}_{57}} : H_2 \rightarrow \text{GL}_{160}(17)$, $\kappa_{\mathfrak{V}_{40}} : H_2 \rightarrow \text{GL}_{96}(17)$, and $\kappa_{\mathfrak{V}_{41}} : H_2 \rightarrow \text{GL}_{96}(17)$ are the representations of H_2 afforded by the modules \mathfrak{V}_{57} , \mathfrak{V}_{40} , and \mathfrak{V}_{41} , respectively. The other matrices $\mathfrak{q}_2 = \kappa_{\mathfrak{V}}(q_2)$ and $\mathfrak{h}_2 = \kappa_{\mathfrak{V}}(h_2)$ are obtained by a similar manner, by block diagonal joining. Similar idea applies to the other side \mathfrak{W} , too.

$\chi|_{D_2} = \tau|_{\alpha(D_2)}$ means $\mathfrak{V}|_{D_2} \cong \mathfrak{W}|_{\alpha(D_2)} = \mathfrak{W}|_T$. Recall that two representations of a group being isomorphic means that there is some matrix \mathcal{T} such that for every element of the group, the matrix for the element corresponding to one of the two representations is conjugate to the matrix for the same element corresponding to the other representation by \mathcal{T} . However, here we have two isomorphic groups D_2 and T instead of identical groups. So, employing the isomorphism $\alpha : D_2 \rightarrow T$, now we can see that $\mathfrak{V}|_{D_2} \cong \mathfrak{W}|_T$ means that there is some $\mathcal{T}_1 \in \text{GL}_{352}(17)$ such that $\kappa_{\mathfrak{V}|_{D_2}}(g) = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}|_T}(\alpha(g)) \mathcal{T}_1$ for all $g \in D_2$.

Assuming we have such \mathcal{T}_1 , we get

$$\begin{aligned} \mathfrak{p}_2 &= \kappa_{\mathfrak{V}|_{D_2}}(p_2) = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}|_T}(\alpha(p_2)) \mathcal{T}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(p_1) \mathcal{T}_1, \quad \text{and} \\ \mathfrak{q}_2 &= \kappa_{\mathfrak{V}|_{D_2}}(q_2) = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}|_T}(\alpha(q_2)) \mathcal{T}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(q_1) \mathcal{T}_1, \end{aligned}$$

thus $\mathfrak{p}_2 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(p_1) \mathcal{T}_1$ and $\mathfrak{q}_2 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(q_1) \mathcal{T}_1$, as desired in the statement (1). Knowing that such \mathcal{T}_1 should exist, we can apply the Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

`IsIsomorphic(GModule(sub<Y|W(p1), W(q1)>), GModule(sub<Y|V(p2), V(q2)>)),`

which gives the boolean value, which is `true` in this case, and the desired transformation matrix \mathcal{T}_1 . So (1) is done.

By assertion (a) and Corollary 7.2.4 of [12] this transformation matrix \mathcal{T}_1 has to be multiplied by a diagonal matrix \mathcal{S}_1 of $\text{GL}_{352}(17)$. In order to calculate its entries one has to get the composition factors of the restrictions $\chi_i|_{D_2}$ and $\tau_j|_T$ to D_2 and T , respectively. From the fusion and the 3 character tables B.4, B.5 and B.2 follows that:

$$\begin{aligned} \chi_{40}|_{D_2} &= \psi_{27} + \psi_{63}, & \chi_{41}|_{D_2} &= \psi_{26} + \psi_{59}, \\ \chi_{57}|_{D_2} &= \psi_{70} + \psi_{73}, & \text{and} \\ \tau_9|_T &= \psi_{26} + \psi_{63} + \psi_{73}, & \tau_{10}|_T &= \psi_{27} + \psi_{59} + \psi_{70}. \end{aligned}$$

Thus, by applying the Meat-axe Algorithm to $\chi_{40}|_{D_2}, \chi_{41}|_{D_2}, \chi_{57}|_{D_2}$, we get the KD_2 -modules $\mathfrak{U}_{26}, \mathfrak{U}_{27}, \mathfrak{U}_{59}, \mathfrak{U}_{63}, \mathfrak{U}_{70}$, and \mathfrak{U}_{73} of $D_2 \cong T$, such that $\mathfrak{V}_{40}|_{D_2} \cong \mathfrak{U}_{27} \oplus \mathfrak{U}_{63}$, $\mathfrak{V}_{41}|_{D_2} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{59}$, $\mathfrak{V}_{57}|_{D_2} \cong \mathfrak{U}_{70} \oplus \mathfrak{U}_{73}$, $\mathfrak{W}_9|_T \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{73}$, and $\mathfrak{W}_{10}|_T \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{73}$. Then, we know that

$$\mathfrak{V}|_{D_2} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{27} \oplus \mathfrak{U}_{59} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{70} \oplus \mathfrak{U}_{73} \cong \mathfrak{W}|_T.$$

Therefore, there is a transformation matrix $\mathcal{S}_0 \in \text{GL}_{352}(17)$ such that

$$\begin{aligned} \mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(p_2) \mathcal{S}_0 &= \text{diag}(\kappa_{\mathfrak{U}_{26}}(p_2), \kappa_{\mathfrak{U}_{27}}(p_2), \kappa_{\mathfrak{U}_{59}}(p_2), \kappa_{\mathfrak{U}_{63}}(p_2), \kappa_{\mathfrak{U}_{70}}(p_2), \kappa_{\mathfrak{U}_{73}}(p_2)), \quad \text{and} \\ \mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(q_2) \mathcal{S}_0 &= \text{diag}(\kappa_{\mathfrak{U}_{26}}(q_2), \kappa_{\mathfrak{U}_{27}}(q_2), \kappa_{\mathfrak{U}_{59}}(q_2), \kappa_{\mathfrak{U}_{63}}(q_2), \kappa_{\mathfrak{U}_{70}}(q_2), \kappa_{\mathfrak{U}_{73}}(q_2)), \end{aligned}$$

where $\kappa_{\mathfrak{U}_{26}} : D_2 \rightarrow \text{GL}_{16}(17)$, $\kappa_{\mathfrak{U}_{27}} : D_2 \rightarrow \text{GL}_{16}(17)$, $\kappa_{\mathfrak{U}_{59}} : D_2 \rightarrow \text{GL}_{80}(17)$, $\kappa_{\mathfrak{U}_{63}} : D_2 \rightarrow \text{GL}_{80}(17)$, $\kappa_{\mathfrak{U}_{70}} : D_2 \rightarrow \text{GL}_{80}(17)$, and $\kappa_{\mathfrak{U}_{73}} : D_2 \rightarrow \text{GL}_{80}(17)$ are the representations of D_2 afforded by the modules \mathfrak{U}_{26} , \mathfrak{U}_{27} , \mathfrak{U}_{59} , \mathfrak{U}_{63} , \mathfrak{U}_{70} , and \mathfrak{U}_{73} , respectively. This matrix \mathcal{S}_0 can be obtained by applying Parker's isomorphism test to the two modules \mathfrak{V} and $\mathfrak{U}_{26} \oplus \mathfrak{U}_{27} \oplus \mathfrak{U}_{59} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{70} \oplus \mathfrak{U}_{73}$. We can assume

that we started with $\mathcal{S}_0^{-1}\kappa_{\mathfrak{Y}}(p_2)\mathcal{S}_0$, $\mathcal{S}_0^{-1}\kappa_{\mathfrak{Y}}(q_2)\mathcal{S}_0$, and $\mathcal{S}_0^{-1}\kappa_{\mathfrak{Y}}(h_2)\mathcal{S}_0$ as \mathfrak{p}_2 , \mathfrak{q}_2 , and \mathfrak{h}_2 , at the beginning of the statement (c). Hence, \mathfrak{p}_2 and \mathfrak{q}_2 are now assumed to be in the block diagonal form as follows, from the beginning (then \mathfrak{h}_2 would be in a certain block form; although not block diagonal, it still carries the structure of $\mathfrak{Y}_{57} \oplus \mathfrak{Y}_{40} \oplus \mathfrak{Y}_{41}$):

$$\begin{aligned}\mathfrak{p}_2 &= \text{diag}(\kappa_{\mathfrak{M}_{26}}(p_2), \kappa_{\mathfrak{M}_{27}}(p_2), \kappa_{\mathfrak{M}_{59}}(p_2), \kappa_{\mathfrak{M}_{63}}(p_2), \kappa_{\mathfrak{M}_{70}}(p_2), \kappa_{\mathfrak{M}_{73}}(p_2)), \text{ and} \\ \mathfrak{q}_2 &= \text{diag}(\kappa_{\mathfrak{M}_{26}}(q_2), \kappa_{\mathfrak{M}_{27}}(q_2), \kappa_{\mathfrak{M}_{59}}(q_2), \kappa_{\mathfrak{M}_{63}}(q_2), \kappa_{\mathfrak{M}_{70}}(q_2), \kappa_{\mathfrak{M}_{73}}(q_2)).\end{aligned}$$

Now, by Schur's Lemma (Lemma 2.1.8 of [12]) and the degrees of these characters appearing in the restriction pattern, the following linear system in the variables $\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f \in K$ holds, where the variables $\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f$ correspond to $\psi_{26}, \psi_{27}, \psi_{59}, \psi_{63}, \psi_{70}, \psi_{73}$, respectively :

$$\begin{aligned}96 &= 16\sigma_b + 80\sigma_d, & 96 &= 16\sigma_a + 80\sigma_c, & 160 &= 80\sigma_e + 80\sigma_f & \text{ and} \\ 176 &= 16\sigma_a + 80\sigma_d + 80\sigma_f, & 176 &= 16\sigma_b + 80\sigma_c + 80\sigma_e.\end{aligned}$$

This system of equations in K has the solution: $\sigma_c = 8 - 7\sigma_a$, $\sigma_d = 8 - 7\sigma_b$, $\sigma_e = 7\sigma_a - 7\sigma_b + 1$, $\sigma_f = -7\sigma_a + 7\sigma_b + 1$, where σ_a and σ_b run through all nonzero elements of K . Hence the diagonal matrix \mathcal{S}_1 has the form

$$\mathcal{S}_{(\sigma_a, \sigma_b)} = \text{diag}(\sigma_a^{16}, \sigma_b^{16}, [8 - 7\sigma_a]^{80}, [8 - 7\sigma_b]^{80}, [7\sigma_a - 7\sigma_b + 1]^{80}, [-7\sigma_a + 7\sigma_b + 1]^{80})$$

for suitable elements $\sigma_a, \sigma_b \in K$. Recall that \mathfrak{p}_2 and \mathfrak{q}_2 are in the block diagonal form as described above, where the sizes of the blocks are 16, 16, 80, 80, 80, and 80, in this order. Therefore, for any σ_a and σ_b in K , the diagonal matrix $\mathcal{S}_{(\sigma_a, \sigma_b)}$ centralizes the two matrices \mathfrak{p}_2 and \mathfrak{q}_2 .

Let

$$\begin{aligned}\mathfrak{r}_{(\sigma_b, \sigma_d)} &= (\mathcal{T}_1 \mathcal{S}_{(\sigma_b, \sigma_d)})^{-1} \kappa_{\mathfrak{Y}}(x_1) \mathcal{T}_1 \mathcal{S}_{(\sigma_b, \sigma_d)}, \quad \text{and} \\ \mathfrak{y}_{(\sigma_b, \sigma_d)} &= (\mathcal{T}_1 \mathcal{S}_{(\sigma_b, \sigma_d)})^{-1} \kappa_{\mathfrak{Y}}(y_1) \mathcal{T}_1 \mathcal{S}_{(\sigma_b, \sigma_d)}.\end{aligned}$$

Then the field elements σ_a, σ_b have to be chosen so that all the entries on the main diagonal of the matrix $\mathcal{S}_{(\sigma_a, \sigma_b)}$ are nonzero and that the matrix group

$$\mathfrak{H}_{\sigma_a, \sigma_b} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_{\sigma_a, \sigma_b}, \mathfrak{y}_{\sigma_a, \sigma_b}, \mathfrak{h}_2 \rangle$$

satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of [12]. The test used here is the order of $\mathfrak{r}_{\sigma_a, \sigma_b} \mathfrak{h}_2$; the order has to be the order of an element in 2Fi_{22} . Running through all possible pairs $(\sigma_a, \sigma_b) \in F^2$ it follows that this test is only successful for the pair $(\sigma_a, \sigma_b) = (15, 9)$, giving the order of $\mathfrak{r}_{\sigma_a, \sigma_b} \mathfrak{h}_2$ equal to 12.

Now, let $\mathcal{S}_1 = \mathcal{S}_{(15, 9)}$, $\mathfrak{r}_2 = (\mathcal{T}_1 \mathcal{S}_1)^{-1} \kappa_{\mathfrak{Y}}(x_1) \mathcal{T}_1 \mathcal{S}_1$, $\mathfrak{y}_2 = (\mathcal{T}_1 \mathcal{S}_1)^{-1} \kappa_{\mathfrak{Y}}(y_1) \mathcal{T}_1 \mathcal{S}_1$, and $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$. Then we have $\mathfrak{p}_2 = \mathcal{S}_1^{-1} \mathfrak{p}_2 \mathcal{S}_1$, $\mathfrak{q}_2 = \mathcal{S}_1^{-1} \mathfrak{q}_2 \mathcal{S}_1$, since any $\mathcal{S}_{(\sigma_a, \sigma_b)}$ centralizes the two matrices \mathfrak{p}_2 and \mathfrak{q}_2 . This matrix group \mathfrak{H} is a candidate which can satisfy the Sylow 2-subgroup test. In Theorem 4.5, it is shown that \mathfrak{H} indeed satisfies the Sylow 2-subgroup test. So (2) is done.

Finally, since \mathfrak{p}_2 and \mathfrak{q}_2 can be expressed as words in terms of \mathfrak{r}_2 and \mathfrak{y}_2 , we have $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle = \langle \mathfrak{r}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$.

For (3), the matrices are stated in [10].

(d) Same idea and process as in (c) are applied to the compatible pair χ', τ' . As done in (2) of (c), a suitable transformation matrix has to be found. However, in this case, there is no such transformation matrix which makes the final matrix

group to satisfy the Sylow 2-subgroup test of Algorithm 7.4.8 of Step 5(c) of [12]. In other words, there was no \mathcal{S}_1 which makes the order of $\mathfrak{r}_2\mathfrak{h}_2$ to be the order of an element in 2Fi_{22} .

□

Lemma 4.4. *Using a very nice presentation for Fischer's simple group Fi_{22} which is stated by Praeger and Soicher in [13], the finitely presented group $G_n = \langle a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n \rangle$ (here the subscript n stands for "nice") with set $\mathcal{R}(G_n)$ of defining relations*

$$\begin{aligned} a_n^2 &= b_n^2 = c_n^2 = d_n^2 = e_n^2 = f_n^2 = g_n^2 = h_n^2 = i_n^2 = 1, \\ (a_nb_n)^3 &= 1, (b_nc_n)^3 = (c_nd_n)^3 = (d_ne_n)^3 = (e_nf_n)^3 = (f_ng_n)^3 = 1, \\ (a_nc_n)^2 &= (a_nd_n)^2 = (a_ne_n)^2 = (a_nf_n)^2 = (a_ng_n)^2 = (a_nh_n)^2 = (a_ni_n)^2 = 1, \\ (b_nd_n)^2 &= (b_ne_n)^2 = (b_nf_n)^2 = (b_ng_n)^2 = (b_nh_n)^2 = (b_ni_n)^2 = 1, \\ (c_ne_n)^2 &= (c_nf_n)^2 = (c_ng_n)^2 = (c_nh_n)^2 = (c_ni_n)^2 = 1, \\ (d_nf_n)^2 &= (d_ng_n)^2 = (e_ng_n)^2 = (e_nh_n)^2 = (e_ni_n)^2 = 1, \\ (d_nh_n)^3 &= (h_ni_n)^3 = (d_ni_n)^2 = (f_nh_n)^2 = (f_ni_n)^2 = (g_nh_n)^2 = (g_ni_n)^2 = 1, \\ (d_nc_nb_nd_ne_nfn_dnh_ni_n)^{10} &= (a_nb_nc_nd_ne_nfn_h_n)^9 = (b_nc_nd_ne_nfn_g_nh_n)^9 = 1 \end{aligned}$$

satisfies the following properties:

- (a) As in page 110 of [13], G_n has a subgroup

$$U_n = \langle a_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n, (a_nb_nc_nd_ne_nh_n)^5 \rangle$$

which is isomorphic to $2.U_6(2)$. G_n has a faithful permutation representation PG_n of degree 3510 with permutation stabilizer U_n .

- (b) As in page 110 of [13], G_n has a subgroup

$$V_n = \langle b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n \rangle$$

which is isomorphic to the (orthogonal) simple group $O_7(3)$. G_n has a faithful permutation representation $(PG_n)'$ of degree 14080 with permutation stabilizer V_n .

- (c) As in page 111 of [13], G_n has a subgroup

$$E_n = \langle a_n, c_n, e_n, g_n, h_n, u_n, v_n, w_n, x_n, y_n, t_n \rangle$$

which is isomorphic to $2^{10} : \mathcal{M}_{22}$, where

$$\begin{aligned} u_n &= b_n a_n c_n b_n, & v_n &= d_n c_n e_n d_n, & w_n &= d_n e_n h_n d_n, \\ x_n &= f_n e_n g_n f_n, & y_n &= d_n c_n h_n d_n, & t_n &= (c_n d_n e_n h_n i_n)^4. \end{aligned}$$

In particular, E_n has order $2^{17} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$.

- (d) E_n has exactly one conjugacy class of 2-central involutions, represented by $z_n = a_n c_n$.
- (e) The centralizer $H_n = C_{G_n}(z_n)$ of z_n in G_n has order $2^{17} \cdot 3^4 \cdot 5$, and $H_n = \langle f_n, g_n, i_n, (a_n b_n c_n)^2, (c_n d_n e_n h_n)^3 \rangle$.
- (f) The intersection $D_n = E_n \cap H_n$ of E_n and H_n has order $2^{17} \cdot 3 \cdot 5$.

Proof. (a) In [13], the presentation is stated without the subscripts n . In page 110 of [13], it is stated that $G_n \cong \text{Fi}_{22}$ and that G_n has a subgroup $U_n \cong 2.U_6(2)$, with generators as written in the statement. By means of MAGMA's command `CosetAction(G_n, U_n)`, we can check that G_n has a permutation representation PG_n of degree 3510 with stabilizer U_n , with order $|PG_n| = 2^{17} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$. As proved in [9], we have $G_n \cong \text{Fi}_{22}$, and therefore PG_n is a faithful permutation representation of G_n .

(b) In page 110 of [13], it is stated that $G_n \cong \text{Fi}_{22}$ has a subgroup $V_n \cong O_7(3)$, with generators as written in the statement. By means of MAGMA's command `CosetAction(G_n, V_n)`, we can check that G_n has a permutation representation $(PG_n)'$ of degree 14080 with stabilizer V_n .

The statement (c) is as written in page 111 of [13].

(d) The MAGMA command `Classes(E_n)` tells us that E_n has exactly one conjugacy class of 2-central involutions, and it also gives a representative z_n for the class. Now, the command `LookupWord(E_n, z_n : ConjugateCheck:=true)` in MAGMA gives an answer $a_n c_n$, which is conjugate to z_n . So, we can redefine z_n to be $z_n = a_n c_n$, and use it as the representative for this class of 2-central involutions.

(e) Let $z_n = a_n c_n$ as in (d). Then, the group $H_n = C_{G_n}(z_n)$ is verified by means of MAGMA to have order $2^{17} \cdot 3^4 \cdot 5$. Now, by applying `GetShortGens` the author obtained the five generators $f_n, g_n, i_n, (a_n b_n c_n)^2, (c_n d_n e_n h_n)^3$ for H_n .

(f) The statement is verified by means of MAGMA. □

In the following theorem the author obtains the presentation for 2Fi_{22} as generators and relations, and establishes an isomorphism from \mathfrak{H} to 2Fi_{22} .

Theorem 4.5. *Keep the notations of Lemma 4.4 and Proposition 4.1 and 4.2. The following statements hold:*

- (a) $D/\langle z_1 \rangle = D_1 \cong E_n$ and $H_2/\langle k_{15} \rangle = H_1 \cong H_n$.
- (b) Let $\phi_1 : D \rightarrow D/\langle z_1 \rangle = D_1$ and $\phi_2 : H_2 \rightarrow H_2/\langle k_{15} \rangle = H_1$ be the canonical epimorphisms. There exist an element $r_2 \in G_n$ and isomorphisms $\varphi_1 : D_1 \rightarrow E_n$ and $\varphi_2 : H_1 \rightarrow (H_n)^{r_2}$ such that $(\varphi_1 \circ \phi_1)(p_1) = p_3 = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = q_3 = (\varphi_2 \circ \phi_2)(q_2)$, and $E_n \cap (H_n)^{r_2} = \langle p_3, q_3 \rangle$ has order $2^{17} \cdot 3 \cdot 5$.
- (c) Let $x_3 = (\varphi_1 \circ \phi_1)(x_1)$, $y_3 = (\varphi_1 \circ \phi_1)(y_1)$, and $h_3 = (\varphi_2 \circ \phi_2)(h_2)$. Then x_3, y_3 and h_3 can be expressed by words in $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n$, as follows:

$$\begin{aligned} x_3 &= \beta_1(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n) = (r_{n,1} r_{n,2} r_{n,3})^3, \\ y_3 &= \beta_2(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n) \\ &= (s_{n,1} s_{n,2} s_{n,4} s_{n,1} s_{n,4} s_{n,2} s_{n,4})^5 (t_{n,1} t_{n,3}^2 t_{n,1} t_{n,3} t_{n,1} t_{n,2} t_{n,1} t_{n,3})^{20}, \\ h_3 &= \beta_3(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n) = (v_{n,1} v_{n,2})^4, \end{aligned}$$

where

$$\begin{aligned} r_{n,1} &= (w_n u_n t_n u_n x_n t_n x_n t_n)^4, & r_{n,2} &= (w_n t_n w_n u_n w_n x_n t_n x_n)^4, \\ r_{n,3} &= (x_n w_n t_n u_n x_n w_n t_n x_n)^2, & r_{n,4} &= (w_n u_n t_n x_n w_n t_n u_n x_n t_n)^4, \\ s_{n,1} &= u_n w_n u_n, & s_{n,2} &= (u_n w_n x_n)^2, & s_{n,3} &= (u_n w_n t_n u_n)^2, \\ s_{n,4} &= (u_n t_n w_n x_n)^4, & t_{n,1} &= s_{n,2} s_{n,4} s_{n,2}^2, \\ t_{n,2} &= (s_{n,4} s_{n,2} s_{n,1} s_{n,3} s_{n,4})^2, & t_{n,3} &= (s_{n,4} s_{n,2} s_{n,4} s_{n,2} s_{n,1} s_{n,3})^2, \\ j_n &= (c_n d_n e_n h_n)^3, & k_n &= (c_n d_n e_n f_n g_n)^2, & l_n &= (a_n b_n c_n d_n e_n h_n)^5, \\ o_n &= (l_n b_n k_n j_n b_n i_n)^6, & p_n &= (j_n k_n j_n l_n b_n i_n j_n i_n)^5, \\ v_{n,1} &= (p_n o_n j_n o_n k_n)^4, & v_{n,2} &= (j_n k_n o_n k_n^2 p_n)^4, & v_{n,3} &= k_n j_n o_n j_n k_n^2 p_n. \end{aligned}$$

- (d) $G_n = \langle x_3, y_3, h_3 \rangle$.

- (e) $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n$ can be expressed as words in x_3, y_3, h_3 , denoted by

$$\begin{aligned} a_n &= \omega_a(x_3, y_3, h_3), & b_n &= \omega_b(x_3, y_3, h_3), & c_n &= \omega_c(x_3, y_3, h_3), \\ d_n &= \omega_d(x_3, y_3, h_3), & e_n &= \omega_e(x_3, y_3, h_3), & f_n &= \omega_f(x_3, y_3, h_3), \\ g_n &= \omega_g(x_3, y_3, h_3), & h_n &= \omega_h(x_3, y_3, h_3), & i_n &= \omega_i(x_3, y_3, h_3). \end{aligned}$$

The explicit words are

$$\begin{aligned} \omega_a(x_3, y_3, h_3) &= (x_3 y_3 x_3)^7, & \omega_b(x_3, y_3, h_3) &= (h_3 y_3 h_3 y_3 h_3 y_3^3 h_3^2 y_3^3 h_3 y_3)^7, \\ \omega_c(x_3, y_3, h_3) &= (y_3^2 x_3 y_3 x_3 y_3^3)^5, \\ \omega_d(x_3, y_3, h_3) &= (h_3 y_3 h_3^2 y_3 h_3 y_3 h_3 y_3 h_3 y_3^2 h_3^2)^{15}, \\ \omega_e(x_3, y_3, h_3) &= (y_3 x_3 y_3^5 x_3)^5, \\ \omega_f(x_3, y_3, h_3) &= (y_3 h_3 y_3 h_3^2 y_3 h_3^2 y_3^2 h_3 y_3^4 h_3^2)^5, \\ \omega_g(x_3, y_3, h_3) &= (x_3 y_3^2 x_3 y_3^3 x_3)^7, & \omega_h(x_3, y_3, h_3) &= (y_3^5 x_3 y_3 x_3)^5, \\ \omega_i(x_3, y_3, h_3) &= (h_3^2 y_3^2 h_3 y_3 h_3^2)^7. \end{aligned}$$

- (f) The finitely presented group $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell \rangle$ (here ℓ stands for "lifted") with set $\mathcal{R}(G_\ell)$ of defining relations

$$\begin{aligned} a_\ell^2 &= b_\ell^2 = c_\ell^2 = d_\ell^2 = e_\ell^2 = f_\ell^2 = g_\ell^2 = h_\ell^2 = i_\ell^2 = 1, \\ (a_\ell b_\ell)^3 &= 1, (b_\ell c_\ell)^3 = z_\ell, (c_\ell d_\ell)^3 = (d_\ell e_\ell)^3 = 1, (e_\ell f_\ell)^3 = (f_\ell g_\ell)^3 = z_\ell, \\ (a_\ell c_\ell)^2 &= (a_\ell d_\ell)^2 = (a_\ell e_\ell)^2 = (a_\ell f_\ell)^2 = (a_\ell g_\ell)^2 = (a_\ell h_\ell)^2 = (a_\ell i_\ell)^2 = 1, \\ (b_\ell d_\ell)^2 &= (b_\ell e_\ell)^2 = (b_\ell f_\ell)^2 = (b_\ell g_\ell)^2 = (b_\ell h_\ell)^2 = (b_\ell i_\ell)^2 = 1, \\ (c_\ell e_\ell)^2 &= (c_\ell f_\ell)^2 = (c_\ell g_\ell)^2 = (c_\ell h_\ell)^2 = (c_\ell i_\ell)^2 = 1, \\ (d_\ell f_\ell)^2 &= (d_\ell g_\ell)^2 = (e_\ell g_\ell)^2 = (e_\ell h_\ell)^2 = (e_\ell i_\ell)^2 = 1, \\ (d_\ell h_\ell)^3 &= (h_\ell i_\ell)^3 = (d_\ell i_\ell)^2 = (f_\ell h_\ell)^2 = (f_\ell i_\ell)^2 = (g_\ell h_\ell)^2 = (g_\ell i_\ell)^2 = 1, \\ (d_\ell c_\ell b_\ell d_\ell e_\ell f_\ell d_\ell h_\ell i_\ell)^{10} &= (a_\ell b_\ell c_\ell d_\ell e_\ell f_\ell h_\ell)^9 = (b_\ell c_\ell d_\ell e_\ell f_\ell g_\ell h_\ell)^9 = 1, \\ z_\ell^2 &= (z_\ell, a_\ell) = (z_\ell, b_\ell) = (z_\ell, c_\ell) = (z_\ell, d_\ell) = (z_\ell, e_\ell) = (z_\ell, f_\ell) = 1, \\ (z_\ell, g_\ell) &= (z_\ell, h_\ell) = (z_\ell, i_\ell) = 1 \end{aligned}$$

has a faithful permutation representation PG_ℓ of degree 28160, which is isomorphic to 2Fi_{22} , having stabilizer $\langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell \rangle$.

- (g) Let

$$\begin{aligned} x_0 &= \beta_1(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \\ y_0 &= \beta_2(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \\ h_0 &= \beta_3(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \end{aligned}$$

where the three words β_1, β_2 , and β_3 are the ones obtained in the statement (b). Then $G_\ell = \langle x_0, y_0, h_0 \rangle$.

- (h) There is an isomorphism $\vartheta : \mathfrak{H} \rightarrow G_\ell$ such that $\vartheta(\mathfrak{r}_2) = x_0, \vartheta(\mathfrak{r}_2) = y_0$, and $\vartheta(\mathfrak{h}_2) = h_0$, and \mathfrak{H} is an irreducible subgroup of $\text{GL}_{352}(17)$.

Proof. (a) The presentation of $D_1 = D/\langle z_1 \rangle$ can be obtained from the presentation of D , as stated in Proposition 4.1(e). Let $\phi_1 : D \rightarrow D/\langle z \rangle = D_1$ be the canonical epimorphism. Then, by trying some random short words, we get that a subgroup $\langle \phi_1(y_1)^4, (\phi_1(x_1)\phi_1(y_1)\phi_1(v_1))^2 \rangle \cong \mathcal{M}_{22}$ works as a permutation stabilizer for the group D_1 , and thus the MAGMA command

```
CosetAction(D_1, <phi_1(y_1)^4, (phi_1(x_1) phi_1(y_1) phi_1(v_1))^2>
```

gives the faithful permutation representation of D_1 of degree 1024. We can now use this faithful permutation representation for all MAGMA computations in this theorem. From Lemma 3.4 of [9] we already have a presentation and faithful permutation representation of $H_1 = H_2/\langle k_{15} \rangle$ of degree 1024. Then, using the permutation representations, it is verified with MAGMA commands `IsIsomorphic(D_1, E_n)` and `IsIsomorphic(H_1, H_n)` that $D_1 \cong E_n$ and $H_1 \cong H_n$.

(b) Let $\phi_1 : D \rightarrow D/\langle z_1 \rangle = D_1$ and $\phi_2 : H_2 \rightarrow H_2/\langle k_{15} \rangle = H_1$ be the canonical epimorphisms. As proved in (a), there exist isomorphisms $\varphi_1 : D_1 \rightarrow E_n$ and $\varphi_0 : H_1 \rightarrow H_n$. Next step is to find some inner automorphism $\delta_0 \in \text{Aut}(G_n)$ of G_n such that the new isomorphism $\varphi_2 = \delta_0 \circ \varphi_0 : H_1 \rightarrow \delta_0(H_n)$ satisfies $(\varphi_1 \circ \phi_1)(p_1) = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = (\varphi_2 \circ \phi_2)(q_2)$. Let φ_1 and φ_0 be fixed. For convenience, let $p_3 = (\varphi_1 \circ \phi_1)(p_1)$, $q_3 = (\varphi_1 \circ \phi_1)(q_1)$, $p'_3 = (\varphi_0 \circ \phi_2)(p_2)$, and $q'_3 = (\varphi_0 \circ \phi_2)(q_2)$.

By means of MAGMA command `IsConjugate(G_n, p'_3, p_3)`, we obtain an element $r_1 \in G_n$ such that $r_1^{-1}p'_3r_1 = p_3$. Fix this r_1 . By means of MAGMA, it is checked that $C_{G_n}(p'_3)$ has order 3072, which is relatively small. Then it is verified by the MAGMA command `exists` that there is some element r_3 of $C_{G_n}(p'_3)$ such that $(r_3r_1)^{-1}q'_3(r_3r_1) = q_3$.

Thus we also have $(r_3r_1)^{-1}p'_3(r_3r_1) = r_1^{-1}(r_3^{-1}p'_3r_3)r_1 = r_1^{-1}p'_3r_1 = p_3$. Therefore, letting $r_2 = r_3r_1$ yields $r_2^{-1}p'_3r_2 = p_3$ and $r_2^{-1}q'_3r_2 = q_3$. Now, let $\delta_0 \in \text{Aut}(G_n)$ be the inner automorphism defined as conjugation by r_2 . Then δ_0 restricted to H_n is an isomorphism $\delta_{0|H_n} : H_n \rightarrow (H_n)^{r_2}$. Thus, $\varphi_2 : H_1 \rightarrow (H_n)^{r_2}$ defined by $\varphi_2 = \delta_{0|H_n} \circ \varphi_0$ is an isomorphism, and we can observe that

$$\begin{aligned} (\varphi_2 \circ \phi_2)(p_2) &= (\delta_{0|H_n} \circ \varphi_0 \circ \phi_2)(p_2) = \delta_{0|H_n}(p'_3) = r_2^{-1}p'_3r_2 = p_3, \quad \text{and} \\ (\varphi_2 \circ \phi_2)(q_2) &= (\delta_{0|H_n} \circ \varphi_0 \circ \phi_2)(q_2) = \delta_{0|H_n}(q'_3) = r_2^{-1}q'_3r_2 = q_3, \end{aligned}$$

and therefore $(\varphi_1 \circ \phi_1)(p_1) = p_3 = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = q_3 = (\varphi_2 \circ \phi_2)(q_2)$, in short. It is checked by means of MAGMA that $E_n \cap (H_n)^{r_2} = \langle p_3, q_3 \rangle$, and that $\langle p_3, q_3 \rangle$ is of order $2^{17} \cdot 3 \cdot 5$.

(c) Let $\phi_1, \phi_2, \varphi_1, \varphi_2$, and r_2 be as obtained in (b), so that the isomorphisms $\varphi_1 : D_1 \rightarrow E_n$ and $\varphi_2 : H_1 \rightarrow (H_n)^{r_2}$ satisfy $(\varphi_1 \circ \phi_1)(p_1) = p_3 = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = q_3 = (\varphi_2 \circ \phi_2)(q_2)$ and $E_n \cap (H_n)^{r_2} = \langle p_3, q_3 \rangle$. Now, let $x_3 = (\varphi_1 \circ \phi_1)(x_1)$, $y_3 = (\varphi_1 \circ \phi_1)(y_1)$, and $h_3 = (\varphi_2 \circ \phi_2)(h_2)$. Then x_3, y_3 and h_3 can be expressed as words in $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n$, and i_n . Thus there are three words by β_1, β_2 , and β_3 , such that

$$\begin{aligned} x_3 &= \beta_1(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n), \\ y_3 &= \beta_2(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n), \\ h_3 &= \beta_3(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n). \end{aligned}$$

The three words β_1, β_2 , and β_3 are obtained by Strategy 1.22, as follows.

It is checked by means of MAGMA that $C_{E_n}(x_3)$ of x_3 in E_n has order $2^{13} \cdot 3 = 24576$, which is relatively small. Applying Strategy 1.22 to this group $C_{E_n}(x_3)$, the author found a word for x_3 ; the results are $C_{E_n}(x_3) = \langle r_{n,1}, r_{n,2}, r_{n,3}, r_{n,4} \rangle$ and $x_3 = (r_{n,1}r_{n,2}r_{n,3})^3$.

Note that y_3 is of order 14 (since y_1 is). By means of MAGMA, it can be checked that the centralizer $C_{E_n}(y_3^7)$ of the involution y_3^7 in E_n has order $2^{16} \cdot 3^2 \cdot 5 \cdot 7$. Applying Strategy 1.22 to this group $C_{E_n}(y_3^7)$, the author found a word for y_3^7 ; the results are $C_{E_n}(y_3^7) = \langle s_{n,1}, s_{n,2}, s_{n,3}, s_{n,4} \rangle$ and $y_3^7 = (s_{n,1}s_{n,2}s_{n,4}s_{n,1}s_{n,4}s_{n,2}s_{n,4})^5$.

Now, by means of the MAGMA command

```
Subgroups(sub<E_n|s_{n,1},s_{n,2},s_{n,3},s_{n,4}> :Al:=Normal)
```

it can be checked that $C_{E_n}(y_3^7) = \langle E_n \mid s_{n,1}, s_{n,2}, s_{n,3}, s_{n,4} \rangle$ has a unique normal subgroup V_n of order 2^{10} . By means of the MAGMA command

```
HasComplement(C_{E_n}(y_3^7),V_n)
```

we can get a complement C_n of V_n in $C_{E_n}(y_3^7)$. Now, by means of the MAGMA command

```
exists(r_n){r_n:r_n in C_{E_n}(y_3^7) | y_3^2 in (C_n)^{r_2}}
```

we obtain some element $r_n \in C_{E_n}(y_3^7)$ such that $y_3^2 \in (C_n)^{r_2}$. It is also checked by means of MAGMA that $(C_n)^{r_2}$ has order $2^6 \cdot 3^2 \cdot 5 \cdot 7 = 20160$. Now, for such C_n and r_2 , the author applied Strategy 1.22 for $(C_n)^{r_2}$ and found a word for y_3^2 ; the results are $(C_n)^{r_2} = \langle t_{n,1}, t_{n,2}, t_{n,3} \rangle$ and $y_3^2 = (t_{n,1}t_{n,3}^2t_{n,1}t_{n,3}t_{n,1}t_{n,2}t_{n,1}t_{n,3})^5$. Finally, since y_3 is of order 14, we get that

$$\begin{aligned} y_3 &= y_3^{15} = (y_3^7)(y_3^2)^4 \\ &= (s_{n,1}s_{n,2}s_{n,4}s_{n,1}s_{n,4}s_{n,2}s_{n,4})^5 (t_{n,1}t_{n,3}^2t_{n,1}t_{n,3}t_{n,1}t_{n,2}t_{n,1}t_{n,3})^{20}, \end{aligned}$$

so we found a word β_2 .

The application of Strategy 1.22 was not so immediate. The command `GetShortGens` for $(H_n)^{r_2}$ was stopped in the middle by the author, until it gave three elements j_n, k_n , and l_n of $(H_n)^{r_2}$. Now, another application of `GetShortGens` using j_n, k_n, l_n and the original generators for G_n yielded $(H_n)^{r_2} = \langle j_n, k_n, o_n, p_n \rangle$. Note that h_3 is of order 3 (since h_2 is). By means of MAGMA, it can be checked that the normalizer $N_{(H_n)^{r_2}}(\langle h_3 \rangle)$ of h_3 in $(H_n)^{r_2}$ has order $2^8 \cdot 3^4 = 20736$, which is relatively small. Using Strategy 1.22 for $N_{(H_n)^{r_2}}(\langle h_3 \rangle)$ in $(H_n)^{r_2}$ yielded $N_{(H_n)^{r_2}}(\langle h_3 \rangle) = \langle v_{n,1}, v_{n,2}, v_{n,3} \rangle$ and finally $h_3 = (v_{n,1}v_{n,2})^4$.

(d) This can easily be checked by means of MAGMA.

(e) The 9 words can be obtained by the command `LookupWord`.

(f) The objective of this theorem is to verify that the matrix group \mathfrak{H} is isomorphic to 2Fi_{22} , and also get a nice presentation for it. Thus \mathfrak{H} is expected to have Fi_{22} as a quotient. In (c) and (e) we had $G_n = \langle a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n \rangle = \langle x_3, y_3, h_3 \rangle \cong \text{Fi}_{22}$, with words for x_3, y_3, h_3 in terms of $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n$, and vice versa (words for a_n, b_n, \dots, i_n in terms of x_3, y_3, h_3). There, the two quotient groups $D_1 = D/\langle z_1 \rangle$ and $H_1 = H_2/\langle k_{15} \rangle$ are embedded in $G_n \cong \text{Fi}_{22}$ as $\langle p_3, q_3, x_3, y_3 \rangle = \langle x_3, y_3 \rangle$ and $\langle p_3, q_3, h_3 \rangle$, respectively. Recall that D and H_2 are embedded in the matrix group \mathfrak{H} as $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2, \mathfrak{h}_2 \rangle$ and $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{h}_2 \rangle$, respectively.

Let

$$\begin{aligned} \mathfrak{a}_n &= \omega_a(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{b}_n &= \omega_b(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{c}_n &= \omega_c(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), \\ \mathfrak{d}_n &= \omega_d(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{e}_n &= \omega_e(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{f}_n &= \omega_f(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), \\ \mathfrak{g}_n &= \omega_g(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{h}_n &= \omega_h(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), & \mathfrak{i}_n &= \omega_i(\mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2), \end{aligned}$$

where the 9 words $\omega_a, \omega_b, \omega_c, \dots, \omega_i$ are the ones obtained in (e). Recall from Lemma 4.4 that the generators $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n$, and i_n of G_n satisfy the set $\mathcal{R}(G_n)$ of defining relations. Now, let's check if these nine matrices $\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n$ also satisfy the set of relations $\mathcal{R}(G_n)$. For example, we had $(b_n c_n d_n e_n f_n g_n h_n)^9 = 1$ as the very last relation in $\mathcal{R}(G_n)$, and thus we check if $(\mathfrak{b}_n \mathfrak{c}_n \mathfrak{d}_n \mathfrak{e}_n \mathfrak{f}_n \mathfrak{g}_n \mathfrak{h}_n)^9$ is the identity matrix in $\text{GL}_{352}(17)$. By means of

MAGMA, it is easy to check if each of these relations is satisfied by the nine matrices $\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n$.

It turned out that all relations are satisfied, except for the three relations $(b_n c_n)^3$, $(e_n f_n)^3$, and $(f_n g_n)^3$. By means of MAGMA, it is easy to check that $(b_n c_n)^3 = (e_n f_n)^3 = (f_n g_n)^3$. So define a matrix $\mathbf{z}_n = (b_n c_n)^3$. Then, we can check that \mathbf{z}_n has order 2, and commutes with all three matrices $\mathbf{r}_2, \mathbf{\eta}_2$, and \mathbf{h}_2 , and therefore also with the nine matrices $\mathbf{a}_n, \mathbf{b}_n, \dots, \mathbf{i}_n$; thus, \mathbf{z}_n is a central involution of \mathfrak{H} . Now, match the nine matrices $\mathbf{a}_n, \mathbf{b}_n, \dots, \mathbf{i}_n$ with nine new abstract variables $a_\ell, b_\ell, \dots, i_\ell$, respectively, and \mathbf{z}_n with another new variable z_ℓ . Then, the ten matrices $\mathbf{a}_n, \mathbf{b}_n, \dots, \mathbf{i}_n, \mathbf{z}_n$ satisfy the set $\mathcal{R}(G_\ell)$ of relations, as written in the statement.

Now, let's verify that the finitely presented group $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell \rangle$ having $\mathcal{R}(G_\ell)$ as its defining relations is isomorphic to 2Fi_{22} . We can observe in the defining relations $\mathcal{R}(G_\ell)$ that z_ℓ is in the center of G_ℓ , and that z_ℓ is of order 1 or 2, since $z_\ell^2 = 1$. It is then also easy to observe that $G_\ell / \langle z_\ell \rangle$ is isomorphic to G_n , having exactly same relations. Therefore, since we have $G_n \cong \text{Fi}_{22}$, we now know $G_\ell / \langle z_\ell \rangle \cong \text{Fi}_{22}$. Hence, if $\text{Order}(z_\ell) = 2$ implies $G_\ell \cong 2\text{Fi}_{22}$, and $\text{Order}(z_\ell) = 1$ implies $G_\ell \cong \text{Fi}_{22}$. So, if we prove that G_ℓ has a permutation representation of order $= |2\text{Fi}_{22}|$, then we can deduce that G_ℓ is indeed isomorphic to 2Fi_{22} , and that this permutation representation is faithful.

As in Lemma 4.4(b), G_n has a subgroup $V_n = \langle b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n \rangle$ which is isomorphic to the simple group $O(7, 3)$, and that G_n has a faithful permutation representation $(PG_n)'$ of degree 14080 with permutation stabilizer V_n .

The strategy is to lift V_n to V_ℓ inside G_ℓ by $V_\ell = \langle b_\ell \xi_1, c_\ell \xi_2, d_\ell \xi_3, e_\ell \xi_4, f_\ell \xi_5, g_\ell \xi_6, h_\ell \xi_7, i_\ell \xi_8 \rangle$ so that $V_\ell \cong O(7, 3)$, by picking suitable $\xi_1, \xi_2, \dots, \xi_8$, where each of $\xi_1, \xi_2, \dots, \xi_8$ is either 1 or z_ℓ . Since each of $\xi_1, \xi_2, \dots, \xi_8$ is either 1 or z_ℓ , there are $2^8 = 256$ cases to check. For each choice, the author let $V_\ell = \langle b_\ell \xi_1, c_\ell \xi_2, d_\ell \xi_3, e_\ell \xi_4, f_\ell \xi_5, g_\ell \xi_6, h_\ell \xi_7, i_\ell \xi_8 \rangle$, and ran the MAGMA command `CosetAction(G_1, V_1)`. Among the 256 cases, only one case returned a permutation representation of degree 28160, while all others returned that of degree 14080. That single case, namely $\xi_1 = z_\ell, \xi_2 = \xi_3 = \xi_4 = 1, \xi_5 = z_\ell, \xi_6 = \xi_7 = \xi_8 = 1$, is exactly the lifting V_ℓ of V_n which we want.

To summarize, let $V_\ell = \langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell \rangle$. Then, the MAGMA command `CosetAction(G_1, V_1)` gives a permutation representation PG_ℓ of degree 28160, which is of order $2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 = |2\text{Fi}_{22}|$. Thus, as asserted, we have $G_\ell \cong 2\text{Fi}_{22}$, and this permutation representation PG_ℓ of G_ℓ is faithful.

(g) Let

$$\begin{aligned} x_0 &= \beta_1(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \\ y_0 &= \beta_2(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \\ h_0 &= \beta_3(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell), \end{aligned}$$

where the three words β_1, β_2 , and β_3 are the ones we obtained in the statement (b). Then $G_\ell = \langle x_0, y_0, h_0 \rangle$ can be checked by means of MAGMA, using the faithful permutation representation PG_ℓ obtained in (f).

(h) Let the three words β_1, β_2 , and β_3 be the ones we obtained in the statement (b). Then, it can be checked by means of MAGMA that

$$\begin{aligned} \mathbf{r}_2 &= \beta_1(\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n), & \mathbf{\eta}_2 &= \beta_2(\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n), \\ \mathbf{h}_2 &= \beta_3(\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n), \end{aligned}$$

where the nine matrices $\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n$, and \mathbf{i}_n are the ones we obtained in the proof of (f) (as words in $\mathfrak{r}_3, \mathfrak{h}_3, \mathfrak{h}_3$). Therefore $\mathfrak{r}_2, \mathfrak{h}_2$, and \mathfrak{h}_2 are contained in $\langle \mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n \rangle$. Hence, $\langle \mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2 \rangle = \mathfrak{H}$ is contained in $\langle \mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n \rangle$. It is clear that $\langle \mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n \rangle$ is contained in $\langle \mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2 \rangle = \mathfrak{H}$, since the nine matrices $\mathbf{a}_n, \mathbf{b}_n, \dots, \mathbf{h}_n$ are words in $\mathfrak{r}_3, \mathfrak{h}_3, \mathfrak{h}_3$. Thus $\mathfrak{H} = \langle \mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n \rangle$. Recall that $\mathfrak{z}_n = (\mathbf{b}_n \mathbf{c}_n)^3$. Now we have

$$\mathfrak{H} = \langle \mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n, \mathfrak{z}_n \rangle.$$

Observe that these generating ten matrices $\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n, \mathbf{i}_n, \mathfrak{z}_n$ satisfy the relations $R(G_\ell)$. Therefore, the matrix group \mathfrak{H} which is generated by these ten matrices is isomorphic to some quotient group of the finitely presented group G_ℓ which has $R(G_\ell)$ as its defining relations. We proved in (f) that $G_\ell \cong 2\text{Fi}_{22}$, and therefore \mathfrak{H} is isomorphic to some quotient group of 2Fi_{22} . Therefore, \mathfrak{H} is isomorphic to 1, Fi_{22} , or 2Fi_{22} . Notice that \mathfrak{z}_n is an involution (hence not an identity element) which is in the center of this matrix group \mathfrak{H} . Among the three choices 1, Fi_{22} , and 2Fi_{22} , the only group with nontrivial center is 2Fi_{22} . Therefore $\mathfrak{H} \cong 2\text{Fi}_{22}$.

Hence, the natural homomorphism $\vartheta : \mathfrak{H} \rightarrow G_\ell$ given by $\vartheta(\mathbf{a}_n) = a_\ell, \vartheta(\mathbf{b}_n) = b_\ell, \vartheta(\mathbf{c}_n) = c_\ell, \vartheta(\mathbf{d}_n) = d_\ell, \vartheta(\mathbf{e}_n) = e_\ell, \vartheta(\mathbf{f}_n) = f_\ell, \vartheta(\mathbf{g}_n) = g_\ell, \vartheta(\mathbf{h}_n) = h_\ell, \vartheta(\mathbf{i}_n) = i_\ell$, and $\vartheta(\mathfrak{z}_n) = z_\ell$ is an isomorphism. Note that x_0, y_0 , and h_0 are expressed by the three words β_1, β_2 , and β_3 in terms of $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell$, and i_ℓ , and also that $\mathfrak{r}_2, \mathfrak{h}_2$, and \mathfrak{h}_2 are expressed by the three words β_1, β_2 , and β_3 in terms of $\mathbf{a}_n, \mathbf{b}_n, \mathbf{c}_n, \mathbf{d}_n, \mathbf{e}_n, \mathbf{f}_n, \mathbf{g}_n, \mathbf{h}_n$, and \mathbf{i}_n . Since ϑ is an isomorphism, it preserves the multiplication of elements, and therefore we have $\vartheta(\mathfrak{r}_2) = x_0, \vartheta(\mathfrak{h}_2) = y_0$, and $\vartheta(\mathfrak{h}_2) = h_0$.

The irreducibility of \mathfrak{H} is checked by means of the MAGMA command `IsIrreducible`.

□

Corollary 4.6. *Keep the notations in Theorem 4.5. Let $K = \text{GF}(17)$. Let $H := G_\ell = \langle x_0, y_0, h_0 \rangle$, and $D_H := \langle x_0, y_0 \rangle$. Then, H and D_H have orders $2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$ and $2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$, respectively. Then the following statements hold:*

- (a) *Each Sylow 2-subgroup S of H has a unique maximal elementary abelian normal subgroup A of order 2^{11} . Also, $N_H(A) \cong D = C_E(z)$.*
- (b) *There is a Sylow 2-subgroup S such that its maximal elementary abelian normal subgroup A of order 2^{11} satisfies $N_H(A) = \langle x_0, y_0 \rangle = D_H$.*
- (c) *There is an isomorphism $\phi_D : D_H \rightarrow D$ such that $\phi_D(x_0) = x_1$ and $\phi_D(y_0) = y_1$.*
- (d) *A system of representatives w_i of the 114 conjugacy classes of H and the corresponding centralizer orders $|C_H(w_i)|$ are given in Table A.5*
- (e) *The character table of H is given in Table B.3.*

Proof. By Theorem 4.5, the finitely presented group $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell \rangle$ has a faithful permutation representation PG_ℓ of degree 28160, with stabilizer $\langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell \rangle$. We also have that x_0, y_0 , and h_0 are expressed as words in terms of $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell$, and that $G_\ell = \langle x_0, y_0, h_0 \rangle$. Therefore, it makes sense to say $H := G_\ell = \langle x_0, y_0, h_0 \rangle$, and computations are also possible in G_ℓ , by means of the faithful permutation representation of G_ℓ , and the corresponding permutations for x_0, y_0 , and h_0 . The orders of H and $D_H := \langle x_0, y_0 \rangle$ are checked with MAGMA.

(a) By means of the MAGMA commands $\text{S:=SylowSubgroup}(H, 2)$ and

$\text{Subgroups}(\text{S:A1:='Normal'}, \text{IsElementaryAbelian:=true}),$

we obtain a Sylow 2-subgroup S of H , and the unique maximal elementary abelian subgroup A of S of order 2^{11} . By the MAGMA command IsIsomorphic , we can also verify that $N_H(A) \cong D$.

(b) Let S be any Sylow 2-subgroup of D_H , and let A be as in (a). Then, it can be checked by the MAGMA that $N_H(A) = D_H$.

(c) Recall that the matrix group $\langle \mathfrak{r}_2, \mathfrak{h}_2 \rangle$ is a faithful representation of $D = \langle x_1, y_1 \rangle$ (by $x_1 \mapsto \mathfrak{r}_2$ and $y_1 \mapsto \mathfrak{h}_2$), since so is $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2, \mathfrak{h}_2 \rangle$. We showed that $\mathfrak{H} = \langle \mathfrak{r}_2, \mathfrak{h}_2, \mathfrak{h}_2 \rangle$ is a faithful representation of $H = \langle x_0, y_0, h_0 \rangle$ (by $x_0 \mapsto \mathfrak{r}_2$, $y_0 \mapsto \mathfrak{h}_2$, and $h_0 \mapsto \mathfrak{h}_2$). Therefore, $\langle \mathfrak{r}_2, \mathfrak{h}_2 \rangle$ is a faithful representation of $\langle x_0, y_0 \rangle = D_H$ (by $x_0 \mapsto \mathfrak{r}_2$ and $y_0 \mapsto \mathfrak{h}_2$). Therefore there is an isomorphism ϕ_D from D_H to D such that $\phi_D(x_0) = x_1$ and $\phi_D(y_0) = y_1$.

(d) and (e) Checked by means of MAGMA and Kratzer's Algorithm 5.3.18 of [12].

□

5. CONSTRUCTION OF FISCHER'S SIMPLE GROUP Fi_{23}

By Lemma 3.2 and Corollary 4.6 the amalgam $H \leftarrow D \rightarrow E$ constructed in sections 3 and 4 satisfies the main condition of G. Michler's Algorithm 7.4.8 of [12]. Therefore we can apply the Algorithm 7.4.8 of [12] to give here a new existence proof for Fischer's simple group Fi_{23} .

The readers should be aware of the abusive notation used in the following theorem. The symbols $\chi, \tau, \chi_i, \tau_i, \mathfrak{V}, \mathfrak{W}, \mathfrak{V}_i, \mathfrak{W}_i, U_i, S_i, \mathcal{T}$ appearing in the following theorem have nothing to do with those of same notations appearing in Theorem 4.3.

Theorem 5.1. *Keep the notations in Lemma 3.2 and Corollary 4.6. Let $K = \text{GF}(17)$. Using the notations of the character tables of B.3, B.2, and B.1 of H, D, E , the following statements hold:*

- (a) *The smallest degree of a nontrivial pair $(\chi, \tau) \in \text{mfchar}_{\mathbb{C}}(H) \times \text{mfchar}_{\mathbb{C}}(E)$ of compatible characters which divides the group order of Fi_{23} is 782.*
- (b) *There is exactly one compatible pair $(\chi, \tau) \in \text{mfchar}_{\mathbb{C}}(H) \times \text{mfchar}_{\mathbb{C}}(E)$ of degree 782 of the groups $H = \langle D_H, h \rangle$ and $E = \langle D, e \rangle$:*

$$(\chi, \tau) = (\chi_1 + \chi_3 + \chi_4, \tau_1 + \tau_2 + \tau_{10} + \tau_{11})$$

with common restriction

$$\chi|_{D_H} = \tau|_D = \psi_1 + \psi_1 + \psi_2 + \psi_6 + \psi_9 + \psi_{10} + \psi_{15},$$

where irreducible characters with bold face indices denote faithful irreducible characters.

- (c) *Let \mathfrak{V} and \mathfrak{W} be the up-to-isomorphism uniquely determined faithful semi-simple multiplicity-free 782-dimensional modules of H and E over $F = \text{GF}(17)$ corresponding to the compatible pair χ, τ , respectively.*

Let $\kappa_{\mathfrak{V}} : H \rightarrow \text{GL}_{782}(17)$ and $\kappa_{\mathfrak{W}} : E \rightarrow \text{GL}_{782}(17)$ be the representations of H and E afforded by the modules \mathfrak{V} and \mathfrak{W} , respectively.

Let $\mathfrak{x} = \kappa_{\mathfrak{V}}(x_0)$, $\mathfrak{y} = \kappa_{\mathfrak{W}}(y_0)$, $\mathfrak{h} = \kappa_{\mathfrak{V}}(h_0)$ in $\kappa_{\mathfrak{V}}(H) \leq \text{GL}_{782}(17)$. Then the following assertions hold:

$\mathfrak{V}|_{D_H} \cong \mathfrak{W}|_D$, and there is a transformation matrix $\mathcal{T} \in \text{GL}_{782}(17)$ such that

$$\mathfrak{x} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(x) \mathcal{T}, \quad \mathfrak{y} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(y) \mathcal{T}.$$

- (d) *Let $Y = \text{GL}_{782}(17)$, $\mathfrak{D} = \langle \mathfrak{x}, \mathfrak{y} \rangle$, $\mathfrak{H} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h} \rangle$. Let $\mathcal{D} = C_Y(\mathfrak{D})$ and $\mathcal{H} = C_Y(\mathfrak{H})$. Let $\mathfrak{e}_1 = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(e) \mathcal{T}$. Let $\mathfrak{E} = \langle \mathfrak{D}, \mathfrak{e}_1 \rangle$ and $\mathcal{E} = C_Y(\mathfrak{E})$. Then the following statements hold:*

- (1) *There is an isomorphism*

$$\gamma : \mathcal{D} \rightarrow \mathcal{D}_1 = \text{GL}_2(17) \times K^{*5} \leq \text{GL}_7(17).$$

- (2) $\mathcal{H}_1 = \gamma(\mathcal{H})$ *is generated by the three diagonal matrices*
 $a_1 = \text{diag}(3, 1, 1, 1, 1, 1, 1)$, $a_2 = \text{diag}(1, 1, 1, 1, 3, 3, 1)$ *and*
 $a_3 = \text{diag}(1, 3, 3, 3, 1, 1, 3)$.
- (3) $\mathcal{E}_1 = \gamma(\mathcal{E})$ *is generated by the four diagonal matrices*
 $b_1 = a_1$, $b_2 = \text{diag}(1, 3, 3, 1, 1, 1, 1)$,
 $b_3 = \text{diag}(1, 1, 1, 3, 3, 1, 1)$ *and* $b_4 = \text{diag}(1, 1, 1, 1, 1, 3, 3)$.
- (4) \mathcal{D} *has* 321×16 \mathcal{H} - \mathcal{E} *double cosets.*
- (5) *The free product $H *_D E$ of H and E with amalgamated subgroup D has exactly one irreducible 782-dimensional representation over K*

whose Sylow 2-subgroups have the same exponent as the ones of H . It corresponds to the \mathcal{H} - \mathcal{E} double coset representative

$$\mathcal{F} = \text{diag}(u, 1^{21}, 1^{77}, 1^{176}, 1^{176}, 16^{330}) \in \text{GL}_{782}(17) \quad \text{where}$$

$$u = \begin{pmatrix} 1 & 1 \\ 9 & 14 \end{pmatrix}.$$

Let $\mathbf{e} = \mathcal{F}^{-1}\mathbf{e}_1\mathcal{F}$ and $\mathfrak{G} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathbf{e} \rangle$. The proof of this Sylow 2-subgroup test for \mathfrak{G} is split into two parts. The first half is the order test for the elements $\mathfrak{h}\mathbf{e}$ and $\mathfrak{y}\mathfrak{h}$; these two matrices have to have orders of elements in Fi_{23} (it turned out that $\mathfrak{h}\mathbf{e}$ has order 12, and $\mathfrak{y}\mathfrak{h}$ has order 13). The second half of the proof is done in (e) of this theorem.

- (6) The four generating matrices of \mathfrak{G} are documented in [10].
- (e) \mathfrak{G} has a faithful permutation representation $P\mathfrak{G}$ of degree 31671 with stabilizer $\mathfrak{H} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h} \rangle$.
- (f) \mathfrak{G} is a finite simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ with centralizer $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H} \cong H$ of the involution $\mathfrak{z} = (\mathfrak{y}\mathfrak{h})^7$.
- (g) \mathfrak{G} has 98 conjugacy classes $\mathfrak{g}_i^{\mathfrak{G}}$ with representatives \mathfrak{g}_i and centralizer orders $|C_{\mathfrak{G}}(\mathfrak{g}_i)|$ as given in Table A.6.
- (h) The character table of \mathfrak{G} coincides with that of Fi_{23} in the Atlas [4], its p. 178-179.

Proof. (a) The character tables of the groups H, D , and E are stated in the Appendix B.3, B.2, and B.1. In the following we use their notations. Using MAGMA and the character tables of H, D , and E and the fusion of the classes of $D_H \cong D$ in H and D in E , an application of Kratzer's Algorithm 7.3.10 of [12] yields the compatible pair stated in assertion (a), dividing the group order of Fi_{23} ; the group order of Fi_{23} is taken from Atlas [4].

(b) The application of Kratzer's Algorithm 7.3.10 of [12] also shows that the pairs (χ, τ) of (b) is the only compatible pair of degree 782 with respect to the fusion of the D -classes into the H - and into the E -classes.

(c) In order to construct the faithful irreducible representation \mathfrak{W} corresponding to the character $\chi = \chi_1 + \chi_3 + \chi_4$ of degree 782, the author employed the MAGMA command `LowIndexSubgroups(PH, 4000)` using the faithful permutation representation PH of H of degree 28160 (see statements (f) and (g) of Theorem 4.5 for construction of PH). MAGMA found a subgroup U_1 of H such that the followings hold.

U_1 is of index 3510 in H , and χ_4 (dimension 429) is a constituent of the permutation character $(1_{U_1})^H$. By `GetShortGens` the author obtained $U_1 = \langle (y_0)^7, (y_0 h_0 y_0^2 h_0)^7, (y_0 h_0 y_0 h_0 y_0)^7, (h_0 y_0^2 h_0 y_0)^7, (h_0 y_0 h_0 y_0^2)^7, (y_0^3 h_0^2 y_0^2)^{11} \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_U)^H$ the author obtained the irreducible KH -module \mathfrak{W}_4 corresponding to χ_4 .

Note that χ_1 is the trivial character. Notice also that χ_3 is the unique irreducible character of H of degree 352, and that we already have an irreducible representation of H of degree 352, namely $\mathfrak{H} = \langle \mathfrak{r}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$. Thus, we can use \mathfrak{H} as a representation of H corresponding to χ_3 .

In order to construct the faithful irreducible representation \mathfrak{W} corresponding to the character $\tau = \tau_1 + \tau_2 + \tau_{10} + \tau_{11}$ of degree 782, the author employed the MAGMA command `LowIndexSubgroups(PE, 1500)` using the faithful permutation

presentation PE of E of degree 1012 (see Lemma 3.2 for construction of PE). MAGMA found subgroups S_1, S_2 , and S_3 such that the followings hold.

S_1 is of index 506 in E , and τ_2 (dimension 22) and τ_{10} (dimension 253) are constituents of the permutation character $(1_{S_1})^E$. By `GetShortGens` we get $S_1 = \langle (y_1^3 e_1 y_1)^3, (y_1^2 x_1 y_1 x_1 y_1)^3, (y_1^4 e_1 x_1)^4, (y_1 e_1 x_1 y_1^3)^4 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_1})^E$ the author obtained the irreducible KE -modules \mathfrak{W}_2 and \mathfrak{W}_{10} corresponding to τ_2 and τ_{10} , respectively.

S_2 is of index 1012 in E , and τ_{11} (dimension 506) is a constituent of the permutation character $(1_{S_2})^E$. By `GetShortGens` we get $S_2 = \langle (x_1 e_1 y_1)^4, (x_1 y_1^3 x_1 e_1)^7, (y_1^2 x_1 y_1 x_1 y_1)^2 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_2})^E$ the author obtained the irreducible KE -module \mathfrak{W}_{11} corresponding to τ_{11} .

Therefore we can obtain the representations $\kappa_{\mathfrak{W}} : H \rightarrow \mathrm{GL}_{782}(17)$ and $\kappa_{\mathfrak{W}} : E \rightarrow \mathrm{GL}_{782}(17)$ of H and E afforded by the modules \mathfrak{W} and \mathfrak{W} , respectively, as follows. As before, $\mathfrak{W} = \mathfrak{W}_1 \oplus \mathfrak{W}_3 \oplus \mathfrak{W}_4$, where \mathfrak{W}_1 is the trivial KH -module, and \mathfrak{W}_3 corresponds to the representation $\kappa_{\mathfrak{W}_3} : H \rightarrow \mathrm{GL}_{352}(17)$ given by $\kappa_{\mathfrak{W}_3}(x_0) = \mathfrak{r}_2$, $\kappa_{\mathfrak{W}_3}(y_0) = \mathfrak{h}_2$, and $\kappa_{\mathfrak{W}_3}(h_0) = \mathfrak{h}_2$. Then, the matrices for $\kappa_{\mathfrak{W}}$ can be obtained by diagonal joining, for example,

$$\kappa_{\mathfrak{W}}(x_0) = \mathrm{diag}(\kappa_{\mathfrak{W}_1}(x_0), \kappa_{\mathfrak{W}_3}(x_0), \kappa_{\mathfrak{W}_4}(x_0)),$$

where $\kappa_{\mathfrak{W}_1} : H \rightarrow \mathrm{GL}_1(17)$ and $\kappa_{\mathfrak{W}_4} : H \rightarrow \mathrm{GL}_{429}(17)$ are the representations of H afforded by the KH -modules \mathfrak{W}_1 and \mathfrak{W}_4 , respectively. And similarly for y_0 and h_0 , too. For E side, let $\mathfrak{W} = \mathfrak{W}_1 \oplus \mathfrak{W}_2 \oplus \mathfrak{W}_{10} \oplus \mathfrak{W}_{11}$, where \mathfrak{W}_1 is the trivial KE -module. Then, the matrices for $\kappa_{\mathfrak{W}}$ can be obtained by diagonal joining, for example,

$$\kappa_{\mathfrak{W}}(x_1) = \mathrm{diag}(\kappa_{\mathfrak{W}_1}(x_1), \kappa_{\mathfrak{W}_2}(x_1), \kappa_{\mathfrak{W}_{10}}(x_1), \kappa_{\mathfrak{W}_{11}}(x_1)),$$

where $\kappa_{\mathfrak{W}_1} : E \rightarrow \mathrm{GL}_1(17)$, $\kappa_{\mathfrak{W}_2} : E \rightarrow \mathrm{GL}_{22}(17)$, $\kappa_{\mathfrak{W}_{10}} : E \rightarrow \mathrm{GL}_{253}(17)$, and $\kappa_{\mathfrak{W}_{11}} : E \rightarrow \mathrm{GL}_{506}(17)$ are the representations of E afforded by the KE -modules $\mathfrak{W}_1, \mathfrak{W}_2, \mathfrak{W}_{10}$, and \mathfrak{W}_{11} , respectively. And similarly for y_1 and e_1 , too.

$\chi|_{D_H} = \tau|_D$ means $\mathfrak{W}|_{D_H} \cong \mathfrak{W}|_D$. Recall that two representations of a group being isomorphic means that there is some matrix \mathcal{T} such that for every element of the group, the matrix for the element corresponding to one of the two representations is conjugate to the matrix for the same element corresponding to the other representation by \mathcal{T} . However, here we have two isomorphic groups D_H and D instead of identical groups. So, employing an isomorphism $\phi_D : D_H \rightarrow D$ as obtained in Corollary 4.6(c), now we can see that $\mathfrak{W}|_{D_H} \cong \mathfrak{W}|_D$ means that there is some $\mathcal{T} \in \mathrm{GL}_{782}(17)$ such that $\kappa_{\mathfrak{W}|_{D_H}}(\phi_D(g)) = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}|_D}(g) \mathcal{T}_1$ for all $g \in D_H$.

Assuming we have such \mathcal{T} , we get

$$\begin{aligned} \mathfrak{r} &= \kappa_{\mathfrak{W}|_{D_H}}(x_0) = \kappa_{\mathfrak{W}|_{D_H}}(\phi_D(x_1)) = \mathcal{T}^{-1} \kappa_{\mathfrak{W}|_D}(x_1) \mathcal{T}, \quad \text{and} \\ \mathfrak{h} &= \kappa_{\mathfrak{W}|_{D_H}}(y_0) = \kappa_{\mathfrak{W}|_{D_H}}(\phi_D(y_1)) = \mathcal{T}^{-1} \kappa_{\mathfrak{W}|_D}(y_1) \mathcal{T}, \end{aligned}$$

thus $\mathfrak{r} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(x_1) \mathcal{T}$ and $\mathfrak{h} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(y_1) \mathcal{T}$, as desired in the statement (1). Knowing that such \mathcal{T} exists, we can apply the Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

`IsIsomorphic(GModule(sub<Y|W(x1), W(y1)>), GModule(sub<Y|V(x0), V(y0)>)),`

which gives the boolean value, which is `true` in this case, and the desired transformation matrix \mathcal{T} .

By assertion (b) and Corollary 7.2.4 of [12] this transformation matrix \mathcal{T} has to be multiplied by a block diagonal matrix \mathcal{S}_0 of $\mathrm{GL}_{782}(17)$. In order to calculate its

entries one has to get the composition factors of the restrictions $\chi_{i|D_H}$ and $\tau_{j|D}$ to D_H and D , respectively. From the fusion and the 3 character tables B.3, B.2 and B.1 follows that:

$$\begin{aligned} \chi_{1|D_H} &= \psi_1, & \chi_{3|D_H} &= \psi_9 + \psi_{10}, & \chi_{4|D_H} &= \psi_1 + \psi_2 + \psi_6 + \psi_{15}, & \text{and} \\ \tau_{1|D} &= \psi_1, & \tau_{2|D} &= \psi_1 + \psi_2, & \tau_{10|D} &= \psi_6 + \psi_9, & \tau_{11|D} &= \psi_{10} + \psi_{15}. \end{aligned}$$

Thus, by applying the Meat-axe Algorithm to $\chi_{4|D_H}$, we get the KD_H -modules $\mathfrak{U}_2, \mathfrak{U}_6$, and \mathfrak{U}_{15} of $D_H \cong D$, such that $\mathfrak{V}_{4|D_H} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_{15}$, where \mathfrak{U}_1 is the trivial KD_H -module. We already have the constituents KD_H -modules \mathfrak{U}_9 and \mathfrak{U}_{10} of the restriction of \mathfrak{V} to D_H , so that $\mathfrak{V}_{3|D_H} \cong \mathfrak{U}_9 \oplus \mathfrak{U}_{10}$. Thus we also have $\mathfrak{W}_{2|D} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_2$, $\mathfrak{W}_{10|D} \cong \mathfrak{U}_6 \oplus \mathfrak{U}_9$, and $\mathfrak{W}_{11|D} \cong \mathfrak{U}_{10} \oplus \mathfrak{U}_{15}$. It is clear that $\mathfrak{V}_{1|D_H} = \mathfrak{U}_1 = \mathfrak{W}_{1|D}$. Notice that the KD_H -modules can be thought of as KD -modules and vice versa, via the isomorphism $\phi_D : D_H \rightarrow D$. Now we have

$$\mathfrak{V}|_{D_H} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_9 \oplus \mathfrak{U}_{10} \oplus \mathfrak{U}_{15} \cong \mathfrak{W}|_D.$$

Therefore, there is a transformation matrix $\mathcal{S}_0 \in \text{GL}_{782}(17)$ such that

$$\mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(x_0) \mathcal{S}_0 = \text{diag}(\kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_2}(x_0), \kappa_{\mathfrak{U}_6}(x_0), \kappa_{\mathfrak{U}_9}(x_0), \kappa_{\mathfrak{U}_{10}}(x_0), \kappa_{\mathfrak{U}_{15}}(x_0)),$$

and

$$\mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(y_0) \mathcal{S}_0 = \text{diag}(\kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_2}(y_0), \kappa_{\mathfrak{U}_6}(y_0), \kappa_{\mathfrak{U}_9}(y_0), \kappa_{\mathfrak{U}_{10}}(y_0), \kappa_{\mathfrak{U}_{15}}(y_0)),$$

where $\kappa_{\mathfrak{U}_1} : D \rightarrow \text{GL}_1(17)$, $\kappa_{\mathfrak{U}_2} : D \rightarrow \text{GL}_{21}(17)$, $\kappa_{\mathfrak{U}_6} : D \rightarrow \text{GL}_{77}(17)$, $\kappa_{\mathfrak{U}_9} : D \rightarrow \text{GL}_{176}(17)$, $\kappa_{\mathfrak{U}_{10}} : D \rightarrow \text{GL}_{176}(17)$, and $\kappa_{\mathfrak{U}_{15}} : D \rightarrow \text{GL}_{330}(17)$ are the representations of D afforded by the modules $\mathfrak{U}_1, \mathfrak{U}_2, \mathfrak{U}_6, \mathfrak{U}_9, \mathfrak{U}_{10}$, and \mathfrak{U}_{15} , respectively. This matrix \mathcal{S}_0 can be obtained by applying Parker's isomorphism test to the two modules \mathfrak{V} and $\mathfrak{U}_1 \oplus \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_9 \oplus \mathfrak{U}_{10} \oplus \mathfrak{U}_{15}$. To be precise, the author did this Parker's isomorphism test first for lower right 781 by 781 submatrices of the relevant matrices, and then enlarged the transformation matrix by diagonally joining 1 in the upper left corner. By this way we can be guaranteed to have right restrictions as we expect. We can assume that we started with $\mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(x_0) \mathcal{S}_0$, $\mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(y_0) \mathcal{S}_0$, and $\mathcal{S}_0^{-1} \kappa_{\mathfrak{V}}(h_0) \mathcal{S}_0$ as \mathfrak{r} , \mathfrak{q} , and \mathfrak{h} , at the beginning of the statement (c). Hence, \mathfrak{r} and \mathfrak{q} are now assumed to be in the block diagonal form as follows, from the beginning (then \mathfrak{h} would be in a certain block form; although not block diagonal, it still carries the structure of $\mathfrak{V}_1 \oplus \mathfrak{V}_3 \oplus \mathfrak{V}_4$):

$$\mathfrak{r} = \text{diag}(\kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_2}(x_0), \kappa_{\mathfrak{U}_6}(x_0), \kappa_{\mathfrak{U}_9}(x_0), \kappa_{\mathfrak{U}_{10}}(x_0), \kappa_{\mathfrak{U}_{15}}(x_0)), \text{ and}$$

$$\mathfrak{q} = \text{diag}(\kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_2}(y_0), \kappa_{\mathfrak{U}_6}(y_0), \kappa_{\mathfrak{U}_9}(y_0), \kappa_{\mathfrak{U}_{10}}(y_0), \kappa_{\mathfrak{U}_{15}}(y_0)).$$

(d) Let $Y = \text{GL}_{782}(17)$, $\mathcal{D} = C_Y(\mathfrak{D})$, $\mathcal{H} = C_Y(\mathfrak{H})$ and $\mathcal{E} = C_Y(\mathfrak{E})$. By (b) the restrictions of the compatible characters (χ, τ) are not multiplicity free. Therefore Theorem 7.2.2 of [12] asserts that one has to determine the $\mathcal{H}\mathcal{E}$ double cosets of \mathcal{D} in order to find all the suitable representations of degree 782 of the free product $H *_D E$ with amalgamated subgroup D . How a double coset representative gives rise to a representation will be explained later in this proof.

For each integer k let v^k denote the diagonal matrix of $\text{GL}_k(17)$ having all diagonal entries equal to $v \in K^*$.

From (c) we know that each element of $\mathfrak{D} = \langle \mathfrak{r}_0, \mathfrak{q}_0 \rangle$ is of the block diagonal form as follows:

$$\mathfrak{g} = \text{diag}(\kappa_{\mathfrak{U}_1}(g), \kappa_{\mathfrak{U}_1}(g), \kappa_{\mathfrak{U}_2}(g), \kappa_{\mathfrak{U}_6}(g), \kappa_{\mathfrak{U}_9}(g), \kappa_{\mathfrak{U}_{10}}(g), \kappa_{\mathfrak{U}_{15}}(g)),$$

for any element $g \in D_H$. Note that all of $\kappa_{\mathcal{M}_1}, \kappa_{\mathcal{M}_2}, \kappa_{\mathcal{M}_6}, \kappa_{\mathcal{M}_9}, \kappa_{\mathcal{M}_{10}}$, and $\kappa_{\mathcal{M}_{15}}$ are irreducible representations of D_H . Therefore, by the Theorem of Artin-Wedderburn (Theorem 2.1.27 of [12]) we know that each element \mathcal{V} of $\mathcal{D} = C_Y(\mathfrak{D})$ can be represented as a blocked diagonal matrix

$$\mathcal{V} = \text{diag}(a, b^{21}, c^{77}, d^{176}, e^{176}, f^{330}) \in \text{GL}_{782}(17)$$

where $a \in \text{GL}_2(17)$ and all $b, c, d, e, f \in K^*$ are uniquely determined by \mathcal{V} .

The map $\gamma : \mathcal{D} \rightarrow \mathcal{D}_1 := \text{GL}_2(17) \times K^{*5}$ defined by

$$\gamma(\mathcal{V}) = \text{diag}(a, b, c, d, e, f) \in \text{GL}_7(17)$$

is an isomorphism. Thus assertion (1) of (d) holds.

Let $\mathcal{H}_1 = \gamma(\mathcal{H})$, $\mathcal{E}_1 = \gamma(\mathcal{E})$. Recall from the proof of (c) that

$$\begin{aligned} \chi_{1|D_H} &= \psi_1, & \chi_{3|D_H} &= \psi_9 + \psi_{10}, & \chi_{4|D_H} &= \psi_1 + \psi_2 + \psi_6 + \psi_{15}, & \text{and} \\ \tau_{1|D} &= \psi_1, & \tau_{2|D} &= \psi_1 + \psi_2, & \tau_{10|D} &= \psi_6 + \psi_9, & \tau_{11|D} &= \psi_{10} + \psi_{15}. \end{aligned}$$

Since $3 \in K$ generates the multiplicative group K^* of K it follows from these restrictions that $\mathcal{H}_1 = \langle a_1, a_2, a_3 \rangle$ and $\mathcal{E}_1 = \langle b_1, b_2, b_3, b_4 \rangle$ where the generators a_i and b_j denote the diagonal matrices of $\text{GL}_7(17)$ given in assertions (2) and (3) of statement (d), respectively. In particular, \mathcal{H}_1 and \mathcal{E}_1 are abelian.

Recall that we have to determine the \mathcal{H} - \mathcal{E} double cosets of \mathcal{D} . Since γ is an isomorphism it suffices to determine the \mathcal{H}_1 - \mathcal{E}_1 double cosets of \mathcal{D}_1 . Let \mathcal{A} and \mathcal{B} be the direct factors $\text{GL}_2(17)$ and K^{*5} of \mathcal{D}_1 , respectively, so $\mathcal{D}_1 = \mathcal{A} \times \mathcal{B}$. Let $\Phi_{\mathcal{A}} : \mathcal{D}_1 \rightarrow \mathcal{A}$ and $\Phi_{\mathcal{B}} : \mathcal{D}_1 \rightarrow \mathcal{B}$ be the projection mappings, i.e. $\Phi_{\mathcal{A}}(ab) = a$ and $\Phi_{\mathcal{B}}(ab) = b$ for all $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Thus, for any element g of \mathcal{D}_1 , the projection $\Phi_{\mathcal{A}}(g)$ is just the upper left 2 by 2 submatrix of g , and $\Phi_{\mathcal{B}}(g)$ the lower right 5 by 5 submatrix of g .

Let λ_1, λ_2 be two representatives for a single \mathcal{H}_1 - \mathcal{E}_1 double coset, i.e. $\mathcal{H}_1\lambda_1\mathcal{E}_1 = \mathcal{H}_1\lambda_2\mathcal{E}_1$. Then $\lambda_2 \in \mathcal{H}_1\lambda_1\mathcal{E}_1$, and therefore $\lambda_2 = a\lambda_1b$ for some $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Note that any element of \mathcal{D}_1 can uniquely be written as multiplication of an element in \mathcal{A} and an element in \mathcal{B} . Therefore, there exist $\xi_1, \delta_1, \theta_1$ in \mathcal{A} and $\xi_2, \delta_2, \theta_2$ in \mathcal{B} such that $\lambda = \theta_1\theta_2$, $a = \xi_1\xi_2$ and $b = \psi_1\delta_2$. Then, $\lambda_2 = a\lambda_1b$ implies $\lambda_2 = (\xi_1\xi_2)(\theta_1\theta_2)(\delta_1\delta_2)$.

Observe that elements of \mathcal{B} commutes both with elements of \mathcal{A} and those of \mathcal{B} . Therefore, $\lambda_2 = (\xi_1\xi_2)(\theta_1\theta_2)(\psi_1\psi_2)$ implies $\lambda_2 = (\xi_1\theta_1\psi_1)(\xi_2\theta_2\psi_2)$. Since $\xi_1\theta_1\psi_1 \in \mathcal{A}$, $\xi_2\theta_2\psi_2 \in \mathcal{B}$, and $\Phi_{\mathcal{B}}(\lambda_1) = \theta_2$, we have $\Phi_{\mathcal{B}}(\lambda_2) = \xi_2\theta_2\psi_2 = \xi_2\Phi_{\mathcal{B}}(\lambda_1)\psi_2$. Let $\mathcal{H}_2 = \Phi_{\mathcal{B}}(\mathcal{H}_1)$ and $\mathcal{E}_2 = \Phi_{\mathcal{B}}(\mathcal{E}_1)$. Then, $\Phi_{\mathcal{B}}(\lambda_2) = \xi_2\Phi_{\mathcal{B}}(\lambda_1)\psi_2$ means that $\Phi_{\mathcal{B}}(\lambda_1)$ and $\Phi_{\mathcal{B}}(\lambda_2)$ represent a single \mathcal{H}_2 - \mathcal{E}_2 double coset in $\mathcal{B} = \Phi_{\mathcal{B}}(\mathcal{D}_1)$.

Since \mathcal{B} is an abelian group, we get that $\mathcal{H}_2\mathcal{E}_2$ is a group, and that \mathcal{H}_2 - \mathcal{E}_2 double cosets are just $\mathcal{H}_2\mathcal{E}_2$ -cosets. Observe that the group $\mathcal{H}_2\mathcal{E}_2$ are generated by $\Phi_{\mathcal{B}}(a_i)$ and $\Phi_{\mathcal{B}}(b_j)$, where

$$\begin{aligned} \Phi_{\mathcal{B}}(a_1) &= \text{Id}(\text{GL}_5(17)), \Phi_{\mathcal{B}}(a_2) = \text{diag}(1, 1, 3, 3, 1), \Phi_{\mathcal{B}}(a_3) = \text{diag}(3, 3, 1, 1, 3), \\ \Phi_{\mathcal{B}}(b_1) &= \text{Id}(\text{GL}_5(17)), \Phi_{\mathcal{B}}(b_2) = \text{diag}(3, 1, 1, 1, 1), \\ \Phi_{\mathcal{B}}(b_3) &= \text{diag}(1, 3, 3, 1, 1), \Phi_{\mathcal{B}}(b_4) = \text{diag}(1, 1, 1, 3, 3). \end{aligned}$$

It is checked in MAGMA that the group $\mathcal{H}_2\mathcal{E}_2$ has order 16^4 , and that the set of sixteen matrices $\zeta_j = \text{diag}(1, 1, 1, 1, j)$, where j runs through $1, 2, 3, \dots, 16$, serves as a complete set of representatives of distinct $\mathcal{H}_2\mathcal{E}_2$ -cosets in \mathcal{B} .

What is just proved is that for any \mathcal{H}_1 - \mathcal{E}_1 double coset representative λ , we can find some $a \in \mathcal{H}_1$ and $b \in \mathcal{E}_1$ such that $\Phi_{\mathcal{B}}(a)\Phi_{\mathcal{B}}(\lambda)\Phi_{\mathcal{B}}(b)$ is of the form ζ_j , for

some j (in the above argument, we had $\xi_2 = \Phi_{\mathcal{B}}(a)$ and $\psi_2 = \Phi_{\mathcal{B}}(b)$). Now, $a\lambda b$ represents the same $\mathcal{H}_1\text{-}\mathcal{E}_1$ double coset as λ , and we now have that $\Phi_{\mathcal{B}}(a\lambda b) = \Phi_{\mathcal{B}}(a)\Phi_{\mathcal{B}}(\lambda)\Phi_{\mathcal{B}}(b)$ is of the form ζ_j . Hence, we can now assume that any double coset representative λ satisfies $\Phi_{\mathcal{B}}(\lambda) = \zeta_j$ for some j . Now, suppose that λ_1 and λ_2 represent a single $\mathcal{H}_1\text{-}\mathcal{E}_1$ double coset. We proved above that $\Phi_{\mathcal{B}}(\lambda_1) = \zeta_j = \Phi_{\mathcal{B}}(\lambda_2)$ for some j .

Fix any $j \in \{1, 2, 3, \dots, 16\}$. Let's find out how many distinct $\mathcal{H}_1\text{-}\mathcal{E}_1$ double coset representative λ there are such that $\Phi_{\mathcal{B}}(\lambda) = \zeta_j$. Let λ_1 and λ_2 be $\mathcal{H}_1\text{-}\mathcal{E}_1$ double coset representatives such that $\Phi_{\mathcal{B}}(\lambda_1) = \zeta_j = \Phi_{\mathcal{B}}(\lambda_2)$ for this fixed j . Then $\lambda_2 \in \mathcal{H}_1\lambda_1\mathcal{E}_1$, and therefore $\lambda_2 = a\lambda_1 b$ for some $a \in \mathcal{H}_1$ and $b \in \mathcal{E}_1$. Observe that \mathcal{H}_1 is abelian and is generated by a_1, a_2, a_3 . Therefore $a = a_1^{\pi_1} a_2^{\pi_2} a_3^{\pi_3}$ for some π_1, π_2, π_3 in $\{1, 2, 3, \dots, 16\}$. Similarly, \mathcal{E}_1 is abelian and is generated by b_1, b_2, b_3, b_4 . Therefore $b = b_1^{\sigma_1} b_2^{\sigma_2} b_3^{\sigma_3} b_4^{\sigma_4}$ for some $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ in $\{1, 2, 3, \dots, 16\}$. We can observe by computation that

$$\begin{aligned} a &= a_1^{\pi_1} a_2^{\pi_2} a_3^{\pi_3} = \text{diag}(3^{\pi_1}, 3^{\pi_3}, 3^{\pi_3}, 3^{\pi_3}, 3^{\pi_2}, 3^{\pi_2}, 3^{\pi_3}), \quad \text{and} \\ b &= b_1^{\sigma_1} b_2^{\sigma_2} b_3^{\sigma_3} b_4^{\sigma_4} = \text{diag}(3^{\sigma_1}, 3^{\sigma_2}, 3^{\sigma_2}, 3^{\sigma_3}, 3^{\sigma_3}, 3^{\sigma_4}, 3^{\sigma_4}). \end{aligned}$$

For $i = 1, 2$, we know that $\lambda_i = \text{diag}(\Phi_{\mathcal{A}}(\lambda_i), \Phi_{\mathcal{B}}(\lambda_i))$ and that $\Phi_{\mathcal{B}}(\lambda_i) = \zeta_j = \text{diag}(1, 1, 1, 1, j)$. Let $\Phi_{\mathcal{A}}(\lambda_1) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ and $\Phi_{\mathcal{A}}(\lambda_2) = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$. Thus, we can observe that

$$\begin{aligned} a\lambda_1 b &= a[\text{diag}(\Phi_{\mathcal{A}}(\lambda_1), \Phi_{\mathcal{B}}(\lambda_1))]b = a[\text{diag}(\begin{pmatrix} A & B \\ C & D \end{pmatrix}, 1, 1, 1, 1, j)]b \\ &= \text{diag}(\begin{pmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{pmatrix}, 3^{\pi_3+\sigma_2}, 3^{\pi_3+\sigma_3}, 3^{\pi_2+\sigma_3}, 3^{\pi_2+\sigma_4}, j3^{\pi_3+\sigma_4}) \end{aligned}$$

We also have

$$\lambda_2 = \text{diag}(\Phi_{\mathcal{A}}(\lambda_2), \Phi_{\mathcal{B}}(\lambda_2)) = \text{diag}(\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}, 1, 1, 1, 1, j),$$

and therefore from $\lambda_2 = a\lambda_1 b$ we get $\begin{pmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$ and $3^{\pi_3+\sigma_2} = 3^{\pi_3+\sigma_3} = 3^{\pi_2+\sigma_3} = 3^{\pi_2+\sigma_4} = 3^{\pi_3+\sigma_4} = 1$. It is easy to see that $3^{\pi_3+\sigma_2} = 3^{\pi_3+\sigma_3} = 3^{\pi_2+\sigma_3} = 3^{\pi_2+\sigma_4} = 3^{\pi_3+\sigma_4} = 1$ implies

$$\pi_3 = -\sigma_2, \quad \pi_3 = -\sigma_3, \quad \pi_2 = -\sigma_3, \quad \pi_2 = -\sigma_4, \quad \pi_3 = -\sigma_4,$$

so that $\pi_2 = \pi_3 = -\sigma_2 = -\sigma_3 = -\sigma_4$.

Now let's look at

$$\begin{pmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}.$$

Since π_1, π_3, σ_1 are arbitrary and $\sigma_2 = -\pi_3$, the above equation can be rewritten as

$$\begin{pmatrix} s & 0 \\ 0 & u \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} t & 0 \\ 0 & u^{-1} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix},$$

where s, t, u are any elements in K^* . By computation we get $\begin{pmatrix} s & 0 \\ 0 & u \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} t & 0 \\ 0 & u^{-1} \end{pmatrix} = \begin{pmatrix} (stA) & (su^{-1}B) \\ (tuC) & D \end{pmatrix}$. Hence, the above equation becomes

$$(*) : \quad \begin{pmatrix} (stA) & (su^{-1}B) \\ (tuC) & D \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}.$$

Define a relation \sim by : $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \sim \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$ if $(*)$ holds. Then, it is easy to prove that \sim is an equivalence relation. It is easy to observe that we have $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \sim \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$ if and only if $D = D'$ and $\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$ can be obtained by multiplying some numbers $\in K^*$ to the first column and the first row of $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. Therefore, it is just an easy computation by hand to get the complete list of representatives for distinct equivalence classes of \sim . The list of representatives is presented as follows.

For cases $D = 0$,

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 1 & 1 \\ c & 0 \end{pmatrix}$$

are the representatives for all possible distinct classes, where c runs through K^* .

This gives $1 + 16 = 17$ cases. For cases $D \neq 0$ and at least one of A, B, C is 0,

$$\begin{pmatrix} 1 & 0 \\ 1 & c \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & c \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & c \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}$$

are the representatives for all possible distinct classes, where c runs through K^* . This gives $4 \times 16 = 64$ cases.

For cases when A, B, C, D are all nonzero,

$$\begin{pmatrix} 1 & 1 \\ c & d \end{pmatrix}$$

are the representatives for all possible distinct classes, where c and d runs through K^* and $c \neq d$. Here we have $16^2 - 16 = 240$ cases. Now we have the complete list of equivalence classes of \sim , and there are $17 + 64 + 240 = 321$ classes.

What is just proved is that, for any fixed $j \in \{1, 2, 3, \dots, 16\}$, there are 321 distinct \mathcal{H}_1 - \mathcal{E}_1 double coset representatives λ such that $\Phi_B(\lambda) = \zeta_j$. Therefore, the number of \mathcal{H}_1 - \mathcal{E}_1 double cosets in \mathcal{D}_1 is exactly $16 \times 321 = 5136$ as desired in (4). We also have a method to enumerate all the double coset representatives; namely, each double coset representative is of the form

$$M_{k,j} = \text{diag}(m_k, 1, 1, 1, 1, j),$$

where j ranges in K^* and $m_k \in \text{GL}_2(17)$ runs through the 321 choices for $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ described above. Since there are 321 choices for m_k , the subscript k can be considered to range in $\{1, 2, \dots, 321\}$. For each k , let $\mathbf{m}_{k,j} = \gamma^{-1}(M_{k,j})$, $\mathbf{e}_{k,j} = \mathbf{m}_{k,j}^{-1} \mathbf{e}_1 \mathbf{m}_{k,j}$, and

$$\mathfrak{G}_{k,j} = \langle \mathfrak{r}, \eta, \mathfrak{h}, \mathbf{e}_{k,j} \rangle.$$

As stated in Theorem 7.2.2 of [12], this matrix group $\mathfrak{G}_{k,j}$ is the representation which corresponds to the double coset representative $\mathbf{m}_{k,j}$.

As mentioned in the statement (5), only first half of the Sylow 2-subgroup test of Step 5(c) of Algorithm 7.4.8 of [12] for each $\mathfrak{G}_{k,j}$ is done here. For each k, j , the author checked the orders of the matrices $\mathfrak{h} \mathbf{e}_{k,j}$ and $\eta \mathfrak{e}_{k,j} \mathfrak{h}$ with MAGMA. If either of the two matrices has an order which can't be the order of an element of Fi_{23} (information on largest order of elements in Fi_{23} is obtained in Atlas [4]), then the particular pair (k, j) is discarded. Through this test, only one case survived, namely the case is with $j = 16$ and $k = k_0$ such that

$$m_{k_0} = \begin{pmatrix} 1 & 1 \\ 9 & 14 \end{pmatrix}.$$

For this k_0 , let $\mathbf{e} = \mathbf{e}_{k_0,16}$ and $\mathfrak{G} = \mathfrak{G}_{k_0,16} = \langle \mathfrak{r}, \eta, \mathfrak{h}, \mathbf{e} \rangle$. It is checked with MAGMA that the two matrices $\mathfrak{h} \mathbf{e}$ and $\eta \mathfrak{e} \mathfrak{h}$ have orders 12 and 13, respectively. So (5) is done. The four generating matrices $\mathfrak{r}, \eta, \mathfrak{h}, \mathbf{e}$ are documented in the author's website [10] as mentioned in (6).

(e) Using the algorithm described in the proof of Theorem 6.2.1 of [12] implemented in MAGMA, a faithful permutation representation $P\mathfrak{G}$ of \mathfrak{G} of degree 31671 with stabilizer \mathfrak{H} is obtained. In order to use this algorithm for getting permutation representation, the author had to look for a short-word matrix in terms of $\mathfrak{r}, \eta, \mathfrak{h}, \mathbf{e}$ which has order 23 (since 23 divides the order of \mathfrak{G} , but not the order of the permutation stabilizer \mathfrak{H}); the matrix $\eta \mathfrak{e} \mathfrak{h}^2$ of order 23 is used.

In particular, it is shown by means of MAGMA that $|\mathfrak{G}| = 2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$, using the faithful permutation representation $P\mathfrak{G}$.

(f) Let $\mathfrak{z} = (\mathfrak{r}\eta^2)^7$. Using the faithful permutation representation $P\mathfrak{G}$ of degree 31671, it is verified that $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H} \cong H$.

(g) Using the faithful permutation representation $P\mathfrak{G}$ of degree 31671 and Kratzer's Algorithm 5.3.18 of [12], the representatives of all the conjugacy classes of \mathfrak{G} are obtained using MAGMA, see Table A.6.

(h) Furthermore, character table of \mathfrak{G} is computed by means of the above permutation representation $P\mathfrak{G}$ and MAGMA. It coincides with the one of Fi_{23} in [4], p. 178 -179. The character table of \mathfrak{G} implies that \mathfrak{G} is a simple group. This completes the proof. □

Remark 5.2. Let $E_1 = V_1 \rtimes \mathcal{M}_{23}$ be the split extension of \mathcal{M}_{23} by its simple module V_1 of dimension 11 over $F = \text{GF}(2)$. Then E_1 has a unique class of 2-central involutions represented by some element z'_1 . However, when applying the Algorithm 2.1 to E_1 , an overgroup of $C_{E_1}(z'_1)$ of odd index satisfying all conditions of Algorithm 2.1 was not found. Hence Algorithm 2.1 can't be applied for further steps.

Remark 5.3. Let $E_2 = V_2 \rtimes \mathcal{M}_{23}$ be the split extension of \mathcal{M}_{23} by its simple module V_2 of dimension 11 over $F = \text{GF}(2)$. Then E_2 has three classes of 2-central involutions represented by some elements z'_2, z'_3 , and z'_4 . However, when applying the Algorithm 2.1 to E_2 , an overgroup of $C_{E_2}(z'_i)$ of odd index satisfying all conditions of Algorithm 2.1 was not found, for any $i = 2, 3, 4$. Hence Algorithm 2.1 can't be applied for further steps.

APPENDIX A. REPRESENTATIVES OF CONJUGACY CLASSES

A.1. Conjugacy classes of $E(\text{Fi}_{23}) = E = \langle x_1, y_1, e_1 \rangle$, with subscripts dropped

Class	Representative	Centralizer	2P	3P	5P	7P	11P	23P
1	1	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	1	1	1	1	1	1
2 ₁	$(y)^7$	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	2 ₁	2 ₁	2 ₁	2 ₁	2 ₁
2 ₂	$(y^3e)^6$	$2^{18} \cdot 3^2 \cdot 5 \cdot 7$	1	2 ₂	2 ₂	2 ₂	2 ₂	2 ₂
2 ₃	$(xye)^8$	$2^{18} \cdot 3^2 \cdot 5$	1	2 ₃	2 ₃	2 ₃	2 ₃	2 ₃
2 ₄	e	$2^{14} \cdot 3 \cdot 7$	1	2 ₄	2 ₄	2 ₄	2 ₄	2 ₄
2 ₅	x	$2^{14} \cdot 3$	1	2 ₅	2 ₅	2 ₅	2 ₅	2 ₅
3	$(y^3e)^4$	$2^7 \cdot 3^2 \cdot 5$	3	1	3	3	3	3
4 ₁	$(xy^2xey^2)^7$	$2^{11} \cdot 3 \cdot 7$	2 ₂	4 ₁	4 ₁	4 ₁	4 ₁	4 ₁
4 ₂	$(xye)^4$	$2^{12} \cdot 3$	2 ₃	4 ₂	4 ₂	4 ₂	4 ₂	4 ₂
4 ₃	$(y^2e)^2$	$2^{12} \cdot 3$	2 ₃	4 ₃	4 ₃	4 ₃	4 ₃	4 ₃
4 ₄	$(y^3e)^3$	$2^{11} \cdot 3$	2 ₂	4 ₄	4 ₄	4 ₄	4 ₄	4 ₄
4 ₅	$(xey)^2$	2^9	2 ₅	4 ₅	4 ₅	4 ₅	4 ₅	4 ₅
4 ₆	$(xyey^3)^2$	2^9	2 ₅	4 ₆	4 ₆	4 ₆	4 ₆	4 ₆
4 ₇	xe	2^8	2 ₄	4 ₇	4 ₇	4 ₇	4 ₇	4 ₇
5	$(xyxye)^2$	$2^3 \cdot 3 \cdot 5$	5	5	1	5	5	5
6 ₁	$(xyxey^2)^5$	$2^7 \cdot 3^2 \cdot 5$	3	2 ₃	6 ₁	6 ₁	6 ₁	6 ₁
6 ₂	$(xyexe)^2$	$2^7 \cdot 3^2$	3	2 ₃	6 ₂	6 ₂	6 ₂	6 ₂
6 ₃	$xyey^3e$	$2^7 \cdot 3^2$	3	2 ₁	6 ₃	6 ₃	6 ₃	6 ₃
6 ₄	$(y^3e)^2$	$2^6 \cdot 3^2$	3	2 ₂	6 ₄	6 ₄	6 ₄	6 ₄
6 ₅	$xyxexey^2$	$2^6 \cdot 3^2$	3	2 ₃	6 ₅	6 ₅	6 ₅	6 ₅
6 ₆	y^4e	$2^5 \cdot 3$	3	2 ₄	6 ₆	6 ₆	6 ₆	6 ₆
6 ₇	$xyxeyxe$	$2^5 \cdot 3$	3	2 ₅	6 ₇	6 ₇	6 ₇	6 ₇
7 ₁	$(y)^2$	$2^3 \cdot 7$	7 ₁	7 ₂	7 ₂	1	7 ₁	7 ₁
7 ₂	$(ye)^2$	$2^3 \cdot 7$	7 ₂	7 ₁	7 ₁	1	7 ₂	7 ₂
8 ₁	$(xye)^2$	2^7	4 ₂	8 ₁	8 ₁	8 ₁	8 ₁	8 ₁
8 ₂	y^2e	2^7	4 ₃	8 ₂	8 ₂	8 ₂	8 ₂	8 ₂
8 ₃	xy^2ey^2e	2^7	4 ₃	8 ₃	8 ₃	8 ₃	8 ₃	8 ₃
8 ₄	xey	2^5	4 ₅	8 ₄	8 ₄	8 ₄	8 ₄	8 ₄
8 ₅	$xyey^3$	2^5	4 ₆	8 ₅	8 ₅	8 ₅	8 ₅	8 ₅
10 ₁	$(xyxey^2)^3$	$2^3 \cdot 3 \cdot 5$	5	10 ₁	2 ₃	10 ₁	10 ₁	10 ₁
10 ₂	xyxye	$2^3 \cdot 5$	5	10 ₂	2 ₂	10 ₂	10 ₂	10 ₂
10 ₃	xyxyey	$2^3 \cdot 5$	5	10 ₃	2 ₁	10 ₃	10 ₃	10 ₃
11 ₁	$(xy^3)^2$	$2 \cdot 11$	11 ₂	11 ₁	11 ₁	11 ₂	1	11 ₁
11 ₂	$(xy^3)^4$	$2 \cdot 11$	11 ₁	11 ₂	11 ₂	11 ₁	1	11 ₂
12 ₁	xyexe	$2^5 \cdot 3$	6 ₂	4 ₂	12 ₁	12 ₁	12 ₁	12 ₁
12 ₂	xyxyxey^2	$2^5 \cdot 3$	6 ₂	4 ₃	12 ₂	12 ₂	12 ₂	12 ₂
12 ₃	y^3e	$2^4 \cdot 3$	6 ₄	4 ₄	12 ₃	12 ₃	12 ₃	12 ₃
12 ₄	y^3ey^2e	$2^4 \cdot 3$	6 ₄	4 ₁	12 ₄	12 ₄	12 ₄	12 ₄
14 ₁	$xyey^2$	$2^3 \cdot 7$	7 ₂	14 ₂	14 ₂	2 ₂	14 ₁	14 ₁
14 ₂	xy^5	$2^3 \cdot 7$	7 ₁	14 ₁	14 ₁	2 ₂	14 ₂	14 ₂
14 ₃	y	$2^2 \cdot 7$	7 ₁	14 ₄	14 ₄	2 ₁	14 ₃	14 ₃
14 ₄	ye	$2^2 \cdot 7$	7 ₂	14 ₃	14 ₃	2 ₁	14 ₄	14 ₄
14 ₅	xyxe	$2^2 \cdot 7$	7 ₂	14 ₆	14 ₆	2 ₄	14 ₅	14 ₅
14 ₆	xy^3xe	$2^2 \cdot 7$	7 ₁	14 ₅	14 ₅	2 ₄	14 ₆	14 ₆
15 ₁	xy^3e	$2 \cdot 3 \cdot 5$	15 ₁	5	3	15 ₂	15 ₂	15 ₁
15 ₂	$(xy^3ey)^2$	$2 \cdot 3 \cdot 5$	15 ₂	5	3	15 ₁	15 ₁	15 ₂
16 ₁	xye	2^5	8 ₁	16 ₁	16 ₂	16 ₂	16 ₁	16 ₂
16 ₂	xey ³	2^5	8 ₁	16 ₂	16 ₁	16 ₁	16 ₂	16 ₁
22 ₁	xy^3	$2 \cdot 11$	11 ₁	22 ₁	22 ₁	22 ₂	2 ₁	22 ₁
22 ₂	xey^2	$2 \cdot 11$	11 ₂	22 ₂	22 ₂	22 ₁	2 ₁	22 ₂
23 ₁	xeyey	23	23 ₁	23 ₁	23 ₂	23 ₂	23 ₂	1
23 ₂	xy^2eyxe	23	23 ₂	23 ₂	23 ₁	23 ₁	23 ₁	1
28 ₁	xy^2xey^2	$2^2 \cdot 7$	14 ₁	28 ₂	28 ₂	4 ₁	28 ₁	28 ₁
28 ₂	xey^3ey	$2^2 \cdot 7$	14 ₂	28 ₁	28 ₁	4 ₁	28 ₂	28 ₂
30 ₁	$xyxey^2$	$2 \cdot 3 \cdot 5$	15 ₁	10 ₁	6 ₁	30 ₂	30 ₂	30 ₁
30 ₂	xy^3ey	$2 \cdot 3 \cdot 5$	15 ₂	10 ₁	6 ₁	30 ₁	30 ₁	30 ₂

A.2. Conjugacy classes of $D(\text{Fi}_{23}) = D = \langle x_1, y_1 \rangle \cong \langle x_0, y_0 \rangle$, with subscripts dropped

Class	Representative	Centralizer	2P	3P	5P	7P	11P
1	1	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	1	1	1	1
2 ₁	$(xy^2)^7$	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	2 ₁	2 ₁	2 ₁	2 ₁
2 ₂	$(y)^7$	$2^{17} \cdot 3^2 \cdot 5 \cdot 7$	1	2 ₂	2 ₂	2 ₂	2 ₂
2 ₃	$(xy^5)^7$	$2^{17} \cdot 3^2 \cdot 5 \cdot 7$	1	2 ₃	2 ₃	2 ₃	2 ₃
2 ₄	$(xyxy^2)^8$	$2^{18} \cdot 3 \cdot 5$	1	2 ₄	2 ₄	2 ₄	2 ₄
2 ₅	$(xyxy^3)^6$	$2^{18} \cdot 3 \cdot 5$	1	2 ₅	2 ₅	2 ₅	2 ₅
2 ₆	$(xyxy^2xy^2)^4$	$2^{16} \cdot 3^2$	1	2 ₆	2 ₆	2 ₆	2 ₆
2 ₇	x	$2^{14} \cdot 3$	1	2 ₇	2 ₇	2 ₇	2 ₇
2 ₈	$(xy^3xy^4)^3$	$2^{14} \cdot 3$	1	2 ₈	2 ₈	2 ₈	2 ₈
2 ₉	$(xy^2xy^3xy^2xy^5)^2$	2^{13}	1	2 ₉	2 ₉	2 ₉	2 ₉
3	$(xyxy^3)^4$	$2^7 \cdot 3^2$	3	1	3	3	3
4 ₁	$(xyxy^3)^3$	$2^{11} \cdot 3$	2 ₅	4 ₁	4 ₁	4 ₁	4 ₁
4 ₂	$(xyxy^3xyxy^4)^3$	$2^{11} \cdot 3$	2 ₅	4 ₂	4 ₂	4 ₂	4 ₂
4 ₃	$(xyxy^2)^4$	2^{12}	2 ₄	4 ₃	4 ₃	4 ₃	4 ₃
4 ₄	$(xyxyxy^2xy^6)^2$	2^{12}	2 ₄	4 ₄	4 ₄	4 ₄	4 ₄
4 ₅	$(xyxy^2xy^2)^2$	$2^{10} \cdot 3$	2 ₆	4 ₅	4 ₅	4 ₅	4 ₅
4 ₆	$(xyxy^2xyxy^6)^3$	$2^{10} \cdot 3$	2 ₆	4 ₆	4 ₆	4 ₆	4 ₆
4 ₇	$xy^7xy^5xy^5$	2^{10}	2 ₅	4 ₇	4 ₇	4 ₇	4 ₇
4 ₈	$(xyxy^2xy^2)^2$	2^9	2 ₇	4 ₈	4 ₈	4 ₈	4 ₈
4 ₉	$(xyxy^2xy^4)^2$	2^9	2 ₇	4 ₉	4 ₉	4 ₉	4 ₉
4 ₁₀	$xy^7xy^5xy^5$	2^8	2 ₈	4 ₁₀	4 ₁₀	4 ₁₀	4 ₁₀
4 ₁₁	$xy^2xy^3xy^2xy^5$	2^8	2 ₉	4 ₁₁	4 ₁₁	4 ₁₁	4 ₁₁
4 ₁₂	$xyxyxy^7xy^4xy^2$	2^8	2 ₉	4 ₁₂	4 ₁₂	4 ₁₂	4 ₁₂
4 ₁₃	$xyxyxyxy^2xy^2$	2^7	2 ₈	4 ₁₃	4 ₁₃	4 ₁₃	4 ₁₃
5	$(xyxy^5)^2$	$2^3 \cdot 5$	5	5	1	5	5
6 ₁	$(xyxy^2xyxy^6)^2$	$2^7 \cdot 3^2$	3	2 ₆	6 ₁	6 ₁	6 ₁
6 ₂	$xy^2xy^2xy^6xy^3$	$2^7 \cdot 3^2$	3	2 ₆	6 ₂	6 ₂	6 ₂
6 ₃	$xyxy^2xy^3xy^2xy^5$	$2^7 \cdot 3^2$	3	2 ₁	6 ₃	6 ₃	6 ₃
6 ₄	$xyxyxy^2xyxyxy^3$	$2^5 \cdot 3^2$	3	2 ₆	6 ₄	6 ₄	6 ₄
6 ₅	$xyxy^2xy^6xy^4$	$2^5 \cdot 3^2$	3	2 ₆	6 ₅	6 ₅	6 ₅
6 ₆	$xyxy^3xyxy^4xy^3$	$2^5 \cdot 3^2$	3	2 ₃	6 ₆	6 ₆	6 ₆
6 ₇	$xyxyxy^4xy^3xyxy^2$	$2^5 \cdot 3^2$	3	2 ₂	6 ₇	6 ₇	6 ₇
6 ₈	$(xyxy^4)^2$	$2^6 \cdot 3$	3	2 ₅	6 ₈	6 ₈	6 ₈
6 ₉	$xyxy^2xy^2$	$2^6 \cdot 3$	3	2 ₄	6 ₉	6 ₉	6 ₉
6 ₁₀	xy^3xy^4	$2^5 \cdot 3$	3	2 ₈	6 ₁₀	6 ₁₀	6 ₁₀
6 ₁₁	xy^4xy^6	$2^5 \cdot 3$	3	2 ₇	6 ₁₁	6 ₁₁	6 ₁₁
7 ₁	$(y)^2$	$2^2 \cdot 7$	7 ₁	7 ₂	7 ₂	1	7 ₁
7 ₂	$(xy^7)^2$	$2^2 \cdot 7$	7 ₂	7 ₁	7 ₁	1	7 ₂
8 ₁	$(xyxy^2)^2$	2^7	4 ₃	8 ₁	8 ₁	8 ₁	8 ₁
8 ₂	$xyxyxy^2xy^6$	2^7	4 ₄	8 ₂	8 ₂	8 ₂	8 ₂
8 ₃	$xyxy^6xy^2xy^4$	2^7	4 ₄	8 ₃	8 ₃	8 ₃	8 ₃
8 ₄	$xyxy^2xy^2$	2^6	4 ₅	8 ₄	8 ₄	8 ₄	8 ₄
8 ₅	$xy^2xy^3xy^3$	2^6	4 ₃	8 ₅	8 ₅	8 ₅	8 ₅
8 ₆	$xy^2xy^2xy^2xy^3$	2^6	4 ₅	8 ₆	8 ₆	8 ₆	8 ₆
8 ₇	$xyxy^3xy^2$	2^5	4 ₈	8 ₇	8 ₇	8 ₇	8 ₇
8 ₈	$xyxy^2xy^4$	2^5	4 ₉	8 ₈	8 ₈	8 ₈	8 ₈
10 ₁	$xyxy^5$	$2^3 \cdot 5$	5	10 ₂	2 ₃	10 ₂	10 ₁
10 ₂	$xyxy^6$	$2^3 \cdot 5$	5	10 ₁	2 ₃	10 ₁	10 ₂
10 ₃	$xyxy^3xy^3$	$2^3 \cdot 5$	5	10 ₃	2 ₅	10 ₃	10 ₃
10 ₄	$xyxy^2xyxy^3$	$2^3 \cdot 5$	5	10 ₆	2 ₂	10 ₆	10 ₄
10 ₅	$xy^2xy^3xy^4$	$2^3 \cdot 5$	5	10 ₅	2 ₄	10 ₅	10 ₅
10 ₆	$xy^2xy^4xy^3$	$2^3 \cdot 5$	5	10 ₄	2 ₂	10 ₄	10 ₆
10 ₇	$xyxyxy^4xy^3xy^2$	$2^3 \cdot 5$	5	10 ₇	2 ₁	10 ₇	10 ₇
11 ₁	$(xy^3)^2$	$2 \cdot 11$	11 ₂	11 ₁	11 ₁	11 ₂	1
11 ₂	$(xy^3)^4$	$2 \cdot 11$	11 ₁	11 ₂	11 ₂	11 ₁	1
12 ₁	$xyxy^2xyxy^6$	$2^5 \cdot 3$	6 ₁	4 ₆	12 ₁	12 ₁	12 ₁
12 ₂	$xyxyxy^2xy^7$	$2^5 \cdot 3$	6 ₁	4 ₅	12 ₂	12 ₂	12 ₂
12 ₃	$xyxy^3$	$2^4 \cdot 3$	6 ₈	4 ₁	12 ₃	12 ₃	12 ₃
12 ₄	$xyxy^3xyxy^4$	$2^4 \cdot 3$	6 ₈	4 ₂	12 ₄	12 ₄	12 ₄
14 ₁	y	$2^2 \cdot 7$	7 ₁	14 ₆	14 ₆	2 ₂	14 ₁
14 ₂	xy^2	$2^2 \cdot 7$	7 ₂	14 ₅	14 ₅	2 ₁	14 ₂
14 ₃	xy^5	$2^2 \cdot 7$	7 ₁	14 ₄	14 ₄	2 ₃	14 ₃
14 ₄	xy^2xy^3	$2^2 \cdot 7$	7 ₂	14 ₃	14 ₃	2 ₃	14 ₄
14 ₅	xy^2xy^4	$2^2 \cdot 7$	7 ₁	14 ₂	14 ₂	2 ₁	14 ₅
14 ₆	xy^3xy^5	$2^2 \cdot 7$	7 ₂	14 ₁	14 ₁	2 ₂	14 ₆
16 ₁	$xyxy^2$	2^5	8 ₁	16 ₁	16 ₂	16 ₂	16 ₁
16 ₂	$xyxyxy^2$	2^5	8 ₁	16 ₂	16 ₁	16 ₁	16 ₂
22 ₁	xy^3	$2 \cdot 11$	11 ₁	22 ₁	22 ₁	22 ₂	2 ₁
22 ₂	$xyxyxy^7xy^2$	$2 \cdot 11$	11 ₂	22 ₂	22 ₂	22 ₁	2 ₁

A.3. Conjugacy classes of $H_2 = \langle p_2, q_2, h_2 \rangle$, with subscripts dropped

Class	Representative	Centralizer	2P	3P	5P
1	1	$2^{18} \cdot 3^4 \cdot 5$	1	1	1
2 ₁	$(pq)^8$	$2^{18} \cdot 3^4 \cdot 5$	1	2 ₁	2 ₁
2 ₂	$(p^2q)^5$	$2^{18} \cdot 3^4 \cdot 5$	1	2 ₂	2 ₂
2 ₃	$(p^2h^2q)^5$	$2^{18} \cdot 3^4 \cdot 5$	1	2 ₃	2 ₃
2 ₄	$(q)^3$	$2^{17} \cdot 3^4 \cdot 5$	1	2 ₄	2 ₄
2 ₅	$(p^2q^2h)^5$	$2^{17} \cdot 3^4 \cdot 5$	1	2 ₅	2 ₅
2 ₆	$(qh)^6$	$2^{16} \cdot 3^2$	1	2 ₆	2 ₆
2 ₇	$(p^2hgh)^6$	$2^{16} \cdot 3^2$	1	2 ₇	2 ₇
2 ₈	$(p^2qp^2h)^3$	$2^{16} \cdot 3^2$	1	2 ₈	2 ₈
2 ₉	$(phphq^2h)^3$	$2^{16} \cdot 3^2$	1	2 ₉	2 ₉
2 ₁₀	$(pqh)^6$	$2^{17} \cdot 3$	1	2 ₁₀	2 ₁₀
2 ₁₁	$(pqhq)^6$	$2^{17} \cdot 3$	1	2 ₁₁	2 ₁₁
2 ₁₂	$(pqp^2h)^6$	$2^{15} \cdot 3^2$	1	2 ₁₂	2 ₁₂
2 ₁₃	$(p^2hq^2)^3$	$2^{15} \cdot 3^2$	1	2 ₁₃	2 ₁₃
2 ₁₄	$(p^2h^2qh)^4$	$2^{16} \cdot 3$	1	2 ₁₄	2 ₁₄
2 ₁₅	$(ph)^5$	$2^{11} \cdot 3^2 \cdot 5$	1	2 ₁₅	2 ₁₅
2 ₁₆	$(phqh)^5$	$2^{11} \cdot 3^2 \cdot 5$	1	2 ₁₆	2 ₁₆
2 ₁₇	$(p)^2$	$2^{13} \cdot 3$	1	2 ₁₇	2 ₁₇
2 ₁₈	$(pq^2)^6$	$2^{13} \cdot 3$	1	2 ₁₈	2 ₁₈
2 ₁₉	$(p^2qp^2qh)^3$	$2^{13} \cdot 3$	1	2 ₁₉	2 ₁₉
2 ₂₀	$(p^2qpqph)^3$	$2^{13} \cdot 3$	1	2 ₂₀	2 ₂₀
2 ₂₁	p^2q^3	$2^{12} \cdot 3$	1	2 ₂₁	2 ₂₁
2 ₂₂	$(pqpq^2h)^3$	$2^{12} \cdot 3$	1	2 ₂₂	2 ₂₂
2 ₂₃	p^3h^2qhqpq	2^{13}	1	2 ₂₃	2 ₂₃
2 ₂₄	$(p^2hph^2)^3$	$2^{10} \cdot 3$	1	2 ₂₄	2 ₂₄
3 ₁	h	$2^8 \cdot 3^4$	3 ₁	1	3 ₁
3 ₂	$(pqh)^4$	$2^9 \cdot 3^3$	3 ₂	1	3 ₂
3 ₃	$(q)^2$	$2^6 \cdot 3^3$	3 ₃	1	3 ₃
4 ₁	$(p^2qph)^6$	$2^{12} \cdot 3^3$	2 ₁	4 ₁	4 ₁
4 ₂	$(phq)^5$	$2^{11} \cdot 3^2 \cdot 5$	2 ₂	4 ₂	4 ₂
4 ₃	$(p^3hgh)^3$	$2^{11} \cdot 3^2 \cdot 5$	2 ₂	4 ₃	4 ₃
4 ₄	$(pq)^4$	$2^{12} \cdot 3$	2 ₁	4 ₄	4 ₄
4 ₅	$(pqh)^3$	$2^{11} \cdot 3$	2 ₁₀	4 ₅	4 ₅
4 ₆	$(q^2h)^3$	$2^{11} \cdot 3$	2 ₁₀	4 ₆	4 ₆
4 ₇	$(pqpqh)^3$	$2^{11} \cdot 3$	2 ₁₀	4 ₇	4 ₇
4 ₈	$(pq^4h)^3$	$2^{11} \cdot 3$	2 ₁₁	4 ₈	4 ₈
4 ₉	$(pqpqhqh)^3$	$2^{11} \cdot 3$	2 ₁₁	4 ₉	4 ₉
4 ₁₀	$(p^2qpq^2h^2)^3$	$2^{11} \cdot 3$	2 ₁₀	4 ₁₀	4 ₁₀
4 ₁₁	$(qh)^3$	$2^{10} \cdot 3$	2 ₆	4 ₁₁	4 ₁₁
4 ₁₂	$(qh^2)^3$	$2^{10} \cdot 3$	2 ₁₀	4 ₁₂	4 ₁₂
4 ₁₃	$(pqhq)^3$	$2^{10} \cdot 3$	2 ₁₁	4 ₁₃	4 ₁₃
4 ₁₄	$(p^2hgh)^3$	$2^{10} \cdot 3$	2 ₇	4 ₁₄	4 ₁₄
4 ₁₅	$(pq^2h^2)^3$	$2^{10} \cdot 3$	2 ₁₀	4 ₁₅	4 ₁₅
4 ₁₆	p^3hph	$2^{10} \cdot 3$	2 ₇	4 ₁₆	4 ₁₆
4 ₁₇	$(p^2hph^2q)^3$	$2^{10} \cdot 3$	2 ₂	4 ₁₇	4 ₁₇
4 ₁₈	$(p^2hphphq)^3$	$2^{10} \cdot 3$	2 ₆	4 ₁₈	4 ₁₈
4 ₁₉	$(p^2hphq)^2$	2^{11}	2 ₁₁	4 ₁₉	4 ₁₉
4 ₂₀	$(pqpqhq)^2$	2^{11}	2 ₁₁	4 ₂₀	4 ₂₀
4 ₂₁	$(pq^2)^3$	$2^9 \cdot 3$	2 ₁₈	4 ₂₁	4 ₂₁
4 ₂₂	$(p^3q)^3$	$2^9 \cdot 3$	2 ₁₇	4 ₂₂	4 ₂₂
4 ₂₃	$(pqp^2h)^3$	$2^9 \cdot 3$	2 ₁₂	4 ₂₃	4 ₂₃
4 ₂₄	$(p^3qh)^3$	$2^9 \cdot 3$	2 ₁₈	4 ₂₄	4 ₂₄
4 ₂₅	$(p^2qhq)^3$	$2^9 \cdot 3$	2 ₁₂	4 ₂₅	4 ₂₅
4 ₂₆	$(p^2hph)^3$	$2^9 \cdot 3$	2 ₁₇	4 ₂₆	4 ₂₆
4 ₂₇	$(p^2h^2qh)^2$	2^{10}	2 ₁₄	4 ₂₇	4 ₂₇
4 ₂₈	p^3qph^2qh	2^{10}	2 ₁₄	4 ₂₈	4 ₂₈
4 ₂₉	p	2^9	2 ₁₇	4 ₂₉	4 ₂₉
4 ₃₀	pq^3	2^9	2 ₁₈	4 ₃₀	4 ₃₀
4 ₃₁	$phqh^2$	2^9	2 ₁₇	4 ₃₁	4 ₃₁
4 ₃₂	p^3qp^2h	2^9	2 ₁₁	4 ₃₂	4 ₃₂
4 ₃₃	$p^2q^2ph^2$	2^9	2 ₁₀	4 ₃₃	4 ₃₃
4 ₃₄	p^2qhghq	2^9	2 ₁₇	4 ₃₄	4 ₃₄
4 ₃₅	p^2hghq^2	2^9	2 ₁₇	4 ₃₅	4 ₃₅
4 ₃₆	$pqpqhph$	2^9	2 ₁₈	4 ₃₆	4 ₃₆
4 ₃₇	pq^2hphqh	2^9	2 ₁₇	4 ₃₇	4 ₃₇
4 ₃₈	p^2hphpq^2h	2^9	2 ₁₇	4 ₃₈	4 ₃₈
4 ₃₉	$pq^2h^2q^2ph$	2^9	2 ₁₀	4 ₃₉	4 ₃₉
4 ₄₀	pq^2phq	2^8	2 ₁₇	4 ₄₀	4 ₄₀
4 ₄₁	$pqpqpqph$	2^8	2 ₁₇	4 ₄₁	4 ₄₁
5	$(ph)^2$	$2^4 \cdot 5$	5	5	1
6 ₁	q^3h	$2^8 \cdot 3^4$	3 ₁	2 ₄	6 ₅

Conjugacy classes of $H_2 = \langle p_2, q_2, h_2 \rangle$, with subscripts dropped (continued)

Class	Representative	Centralizer	2P	3P	5P
6 ₂	$(pqpqh)^3$	$2^8 \cdot 3^4$	3 ₁	2 ₅	6 ₃
6 ₃	$(pq^2ph)^3$	$2^8 \cdot 3^4$	3 ₁	2 ₅	6 ₂
6 ₄	$(phphq)^2$	$2^8 \cdot 3^4$	3 ₁	2 ₁	6 ₄
6 ₅	q^2h^2	$2^8 \cdot 3^4$	3 ₁	2 ₄	6 ₁
6 ₆	$(p^2qhqh)^3$	$2^8 \cdot 3^4$	3 ₁	2 ₃	6 ₆
6 ₇	$(pqhqph)^3$	$2^8 \cdot 3^4$	3 ₁	2 ₂	6 ₇
6 ₈	$(p^2qph)^4$	$2^9 \cdot 3^3$	3 ₂	2 ₁	6 ₈
6 ₉	$(p^3hgh)^2$	$2^9 \cdot 3^3$	3 ₂	2 ₂	6 ₉
6 ₁₀	$p^2hph^2pq^2$	$2^9 \cdot 3^3$	3 ₂	2 ₃	6 ₁₀
6 ₁₁	p^3qhphq	$2^8 \cdot 3^3$	3 ₂	2 ₅	6 ₁₁
6 ₁₂	$p^2hpqhppq$	$2^8 \cdot 3^3$	3 ₂	2 ₄	6 ₁₂
6 ₁₃	$(qh)^2$	$2^8 \cdot 3^2$	3 ₁	2 ₆	6 ₁₃
6 ₁₄	$(pqpqh)^2$	$2^8 \cdot 3^2$	3 ₁	2 ₁₂	6 ₁₆
6 ₁₅	$(p^2hgh)^2$	$2^8 \cdot 3^2$	3 ₁	2 ₇	6 ₁₅
6 ₁₆	$(pq^2ph^2)^2$	$2^8 \cdot 3^2$	3 ₁	2 ₁₂	6 ₁₄
6 ₁₇	$phqph^2$	$2^8 \cdot 3^2$	3 ₁	2 ₁₃	6 ₁₈
6 ₁₈	$phqhph^2$	$2^8 \cdot 3^2$	3 ₁	2 ₁₃	6 ₁₇
6 ₁₉	pq^3h^2phq	$2^8 \cdot 3^2$	3 ₁	2 ₈	6 ₁₉
6 ₂₀	$p^3hpqh^2q^2$	$2^8 \cdot 3^2$	3 ₁	2 ₉	6 ₂₀
6 ₂₁	$(pq^2hq^2)^2$	$2^6 \cdot 3^3$	3 ₃	2 ₂	6 ₂₁
6 ₂₂	$p^2hphppq$	$2^6 \cdot 3^3$	3 ₃	2 ₃	6 ₂₂
6 ₂₃	q^2hghqh	$2^6 \cdot 3^3$	3 ₃	2 ₁	6 ₂₃
6 ₂₄	p^2qh^2	$2^7 \cdot 3^2$	3 ₂	2 ₆	6 ₂₄
6 ₂₅	p^2qp^2h	$2^7 \cdot 3^2$	3 ₂	2 ₈	6 ₂₅
6 ₂₆	p^2hphph	$2^7 \cdot 3^2$	3 ₂	2 ₇	6 ₂₆
6 ₂₇	$q^2h^2qhqh^2$	$2^7 \cdot 3^2$	3 ₂	2 ₉	6 ₂₇
6 ₂₈	q	$2^5 \cdot 3^3$	3 ₃	2 ₄	6 ₂₈
6 ₂₉	p^3qp^2hph	$2^5 \cdot 3^3$	3 ₃	2 ₅	6 ₂₉
6 ₃₀	$(pqh)^2$	$2^8 \cdot 3$	3 ₂	2 ₁₀	6 ₃₀
6 ₃₁	$(pqhq)^2$	$2^8 \cdot 3$	3 ₂	2 ₁₁	6 ₃₁
6 ₃₂	p^2hq^2	$2^6 \cdot 3^2$	3 ₂	2 ₁₃	6 ₃₂
6 ₃₃	$phqhgh^2$	$2^6 \cdot 3^2$	3 ₂	2 ₁₅	6 ₃₃
6 ₃₄	$ph^2q^2h^2$	$2^6 \cdot 3^2$	3 ₂	2 ₁₆	6 ₃₄
6 ₃₅	p^2qpq^2ph	$2^6 \cdot 3^2$	3 ₂	2 ₁₂	6 ₃₅
6 ₃₆	p^3h^2pq	$2^7 \cdot 3$	3 ₂	2 ₁₄	6 ₃₆
6 ₃₇	$qhqh^2$	$2^5 \cdot 3^2$	3 ₃	2 ₆	6 ₃₇
6 ₃₈	p^3ppq	$2^5 \cdot 3^2$	3 ₃	2 ₁₃	6 ₄₁
6 ₃₉	$pqhq^2ph$	$2^5 \cdot 3^2$	3 ₃	2 ₁₂	6 ₄₃
6 ₄₀	$phphq^2h$	$2^5 \cdot 3^2$	3 ₃	2 ₉	6 ₄₀
6 ₄₁	$p^2q^2h^2qh$	$2^5 \cdot 3^2$	3 ₃	2 ₁₃	6 ₃₈
6 ₄₂	$p^2qhphppq$	$2^5 \cdot 3^2$	3 ₃	2 ₇	6 ₄₂
6 ₄₃	$p^2hqphppq$	$2^5 \cdot 3^2$	3 ₃	2 ₁₂	6 ₃₉
6 ₄₄	pqh^2qhph	$2^5 \cdot 3^2$	3 ₃	2 ₈	6 ₄₄
6 ₄₅	p^2h	$2^6 \cdot 3$	3 ₃	2 ₁₇	6 ₄₅
6 ₄₆	$(pq^2)^2$	$2^6 \cdot 3$	3 ₃	2 ₁₈	6 ₄₆
6 ₄₇	p^2qp^2qh	$2^6 \cdot 3$	3 ₃	2 ₁₉	6 ₄₇
6 ₄₈	p^2ppqh	$2^6 \cdot 3$	3 ₃	2 ₂₀	6 ₄₈
6 ₄₉	p^2ppq	$2^4 \cdot 3^2$	3 ₃	2 ₁₅	6 ₄₉
6 ₅₀	p^2qhppq^2	$2^4 \cdot 3^2$	3 ₃	2 ₁₆	6 ₅₀
6 ₅₁	p^2q^3h	$2^5 \cdot 3$	3 ₃	2 ₂₁	6 ₅₁
6 ₅₂	p^2hph^2	$2^5 \cdot 3$	3 ₂	2 ₂₄	6 ₅₂
6 ₅₃	$pqpq^2h$	$2^5 \cdot 3$	3 ₃	2 ₂₂	6 ₅₃
8 ₁	$(p^2qph)^3$	$2^7 \cdot 3$	4 ₁	8 ₁	8 ₁
8 ₂	$(pqhqhq)^3$	$2^7 \cdot 3$	4 ₁	8 ₂	8 ₂
8 ₃	$(pq)^2$	2^7	4 ₄	8 ₃	8 ₃
8 ₄	$pqpqhq$	2^7	4 ₂₀	8 ₄	8 ₄
8 ₅	pqh^2qh	2^7	4 ₂₀	8 ₅	8 ₅
8 ₆	p^3hqph	2^7	4 ₂₀	8 ₆	8 ₆
8 ₇	$pqphphq$	2^7	4 ₂₀	8 ₇	8 ₇
8 ₈	pq^2hq	2^6	4 ₁₁	8 ₈	8 ₈
8 ₉	p^2qpq^2	2^6	4 ₁₁	8 ₉	8 ₉
8 ₁₀	p^2q^2ph	2^6	4 ₁₆	8 ₁₀	8 ₁₀
8 ₁₁	p^2hphq	2^6	4 ₁₉	8 ₁₁	8 ₁₁
8 ₁₂	p^2h^2qh	2^6	4 ₂₇	8 ₁₂	8 ₁₂
8 ₁₃	p^3qphq	2^6	4 ₂₇	8 ₁₃	8 ₁₃
8 ₁₄	$pqpqpqh$	2^6	4 ₁₆	8 ₁₄	8 ₁₄
9	$(pqpqh)^2$	$2^3 \cdot 3^2$	9	3 ₁	9
10 ₁	p^2q	$2^4 \cdot 5$	5	10 ₁	2 ₂
10 ₂	p^2qh	$2^4 \cdot 5$	5	10 ₂	2 ₁
10 ₃	p^2h^2q	$2^4 \cdot 5$	5	10 ₃	2 ₃
10 ₄	ph	$2^3 \cdot 5$	5	10 ₄	2 ₁₅

Conjugacy classes of $H_2 = \langle p_2, q_2, h_2 \rangle$, with subscripts dropped (continued)

Class	Representative	Centralizer	2P	3P	5P
10 ₅	p^2q^2	$2^3 \cdot 5$	5	10 ₅	2 ₄
10 ₆	$phqh$	$2^3 \cdot 5$	5	10 ₆	2 ₁₆
10 ₇	p^2q^2h	$2^3 \cdot 5$	5	10 ₇	2 ₅
12 ₁	$(p^2qph)^2$	$2^7 \cdot 3^2$	6 ₈	4 ₁	12 ₁
12 ₂	$(pqhqh)^2$	$2^7 \cdot 3^2$	6 ₈	4 ₁	12 ₂
12 ₃	p^3hpq	$2^5 \cdot 3^3$	6 ₄	4 ₁	12 ₃
12 ₄	p^3h^2qh	$2^6 \cdot 3^2$	6 ₉	4 ₃	12 ₄
12 ₅	ph^2qh^2	$2^6 \cdot 3^2$	6 ₉	4 ₂	12 ₅
12 ₆	p^2h^2pqhpq	$2^6 \cdot 3^2$	6 ₈	4 ₁	12 ₆
12 ₇	pqh	$2^6 \cdot 3$	6 ₃₀	4 ₅	12 ₇
12 ₈	q^2h	$2^6 \cdot 3$	6 ₃₀	4 ₆	12 ₈
12 ₉	$pqphq$	$2^6 \cdot 3$	6 ₃₀	4 ₇	12 ₉
12 ₁₀	pq^4h	$2^6 \cdot 3$	6 ₃₁	4 ₈	12 ₁₀
12 ₁₁	$pqphpqh$	$2^6 \cdot 3$	6 ₃₁	4 ₉	12 ₁₁
12 ₁₂	$p^2qpq^2h^2$	$2^6 \cdot 3$	6 ₃₀	4 ₁₀	12 ₁₂
12 ₁₃	pq^2hq^2	$2^4 \cdot 3^2$	6 ₂₁	4 ₂	12 ₁₃
12 ₁₄	$pqpqh^2$	$2^4 \cdot 3^2$	6 ₂₁	4 ₃	12 ₁₄
12 ₁₅	qh	$2^5 \cdot 3$	6 ₁₃	4 ₁₁	12 ₁₅
12 ₁₆	qh^2	$2^5 \cdot 3$	6 ₃₀	4 ₁₂	12 ₁₆
12 ₁₇	$pqph$	$2^5 \cdot 3$	6 ₁₄	4 ₂₃	12 ₂₄
12 ₁₈	pqh^2	$2^5 \cdot 3$	6 ₃₁	4 ₁₃	12 ₁₈
12 ₁₉	p^2ghq	$2^5 \cdot 3$	6 ₁₄	4 ₂₅	12 ₂₃
12 ₂₀	p^2h^2qh	$2^5 \cdot 3$	6 ₁₅	4 ₁₄	12 ₂₀
12 ₂₁	pq^2h^2	$2^5 \cdot 3$	6 ₃₀	4 ₁₅	12 ₂₁
12 ₂₂	$phphq$	$2^5 \cdot 3$	6 ₄	4 ₄	12 ₂₂
12 ₂₃	pq^2ph^2	$2^5 \cdot 3$	6 ₁₆	4 ₂₅	12 ₁₉
12 ₂₄	$p^2q^2p^2h$	$2^5 \cdot 3$	6 ₁₆	4 ₂₃	12 ₁₇
12 ₂₅	p^2hph^2q	$2^5 \cdot 3$	6 ₉	4 ₁₇	12 ₂₅
12 ₂₆	p^3qh^2pq	$2^5 \cdot 3$	6 ₁₅	4 ₁₆	12 ₂₆
12 ₂₇	$p^2hphphq$	$2^5 \cdot 3$	6 ₁₃	4 ₁₈	12 ₂₇
12 ₂₈	pq^2	$2^4 \cdot 3$	6 ₄₆	4 ₂₁	12 ₂₈
12 ₂₉	p^3q	$2^4 \cdot 3$	6 ₄₅	4 ₂₂	12 ₂₉
12 ₃₀	p^3qh	$2^4 \cdot 3$	6 ₄₆	4 ₂₄	12 ₃₀
12 ₃₁	p^2hph	$2^4 \cdot 3$	6 ₄₅	4 ₂₆	12 ₃₁
16 ₁	pq	2^5	8 ₃	16 ₁	16 ₂
16 ₂	p^3q^2	2^5	8 ₃	16 ₂	16 ₁
18 ₁	$pqpqh$	$2^3 \cdot 3^2$	9	6 ₂	18 ₃
18 ₂	$pqph^2$	$2^3 \cdot 3^2$	9	6 ₁	18 ₅
18 ₃	pq^2ph	$2^3 \cdot 3^2$	9	6 ₃	18 ₁
18 ₄	p^2qh^2	$2^3 \cdot 3^2$	9	6 ₆	18 ₄
18 ₅	pq^2pqh	$2^3 \cdot 3^2$	9	6 ₅	18 ₂
18 ₆	pqh^2p^2	$2^3 \cdot 3^2$	9	6 ₇	18 ₆
18 ₇	p^3h^2pqh	$2^3 \cdot 3^2$	9	6 ₄	18 ₇
20 ₁	phq	$2^3 \cdot 5$	10 ₁	20 ₁	4 ₂
20 ₂	phq^2hq	$2^3 \cdot 5$	10 ₁	20 ₂	4 ₃
24 ₁	p^2qph	$2^4 \cdot 3$	12 ₁	8 ₁	24 ₁
24 ₂	pqh^2qh	$2^4 \cdot 3$	12 ₂	8 ₂	24 ₂

A.4. Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle \cong \langle p_1, q_1 \rangle$, with subscripts dropped

Class	Representative	Centralizer	2P	3P	5P
1	1	$2^{18} \cdot 3 \cdot 5$	1	1	1
2 ₁	$(pq)^8$	$2^{18} \cdot 3 \cdot 5$	1	2 ₁	2 ₁
2 ₂	$(p^2q)^5$	$2^{18} \cdot 3 \cdot 5$	1	2 ₂	2 ₂
2 ₃	$(p^2qpq^2pq^2)^5$	$2^{18} \cdot 3 \cdot 5$	1	2 ₃	2 ₃
2 ₄	$(q)^3$	$2^{17} \cdot 3 \cdot 5$	1	2 ₄	2 ₄
2 ₅	$(p^3q^2p^2ppq)^3$	$2^{17} \cdot 3 \cdot 5$	1	2 ₅	2 ₅
2 ₆	$(p^2qpq^2)^4$	$2^{16} \cdot 3$	1	2 ₆	2 ₆
2 ₇	$(p^2qpq^4)^3$	$2^{16} \cdot 3$	1	2 ₇	2 ₇
2 ₈	$(p^3qp^2qpqpq)^4$	$2^{16} \cdot 3$	1	2 ₈	2 ₈
2 ₉	$(p^3qpqp^2q^4)^3$	$2^{16} \cdot 3$	1	2 ₉	2 ₉
2 ₁₀	$(p^3qp^2q^4)^3$	2^{17}	1	2 ₁₀	2 ₁₀
2 ₁₁	$(p^2qpq^2pqp^2q^2)^2$	2^{17}	1	2 ₁₁	2 ₁₁
2 ₁₂	$(p^3qpq)^3$	$2^{15} \cdot 3$	1	2 ₁₂	2 ₁₂
2 ₁₃	$(p^3qpqp^2q)^3$	$2^{15} \cdot 3$	1	2 ₁₃	2 ₁₃
2 ₁₄	$(p^3qp^2q^2pq)^4$	2^{16}	1	2 ₁₄	2 ₁₄
2 ₁₅	$(pq^2)^6$	$2^{13} \cdot 3$	1	2 ₁₅	2 ₁₅
2 ₁₆	$(p^3q)^6$	$2^{13} \cdot 3$	1	2 ₁₆	2 ₁₆
2 ₁₇	$(pqpqpqpq^2)^3$	$2^{13} \cdot 3$	1	2 ₁₇	2 ₁₇
2 ₁₈	$(p^3qpq^2pqpq^4)^3$	$2^{13} \cdot 3$	1	2 ₁₈	2 ₁₈
2 ₁₉	$(p^3q^2pqpq)^4$	2^{14}	1	2 ₁₉	2 ₁₉
2 ₂₀	$(p^3q^2pqpq^2)^2$	2^{14}	1	2 ₂₀	2 ₂₀
2 ₂₁	$(pqp^2pq^5)^3$	$2^{12} \cdot 3$	1	2 ₂₁	2 ₂₁
2 ₂₂	$(p^3qpq^2pqpq)^3$	$2^{12} \cdot 3$	1	2 ₂₂	2 ₂₂
2 ₂₃	$(p^2q^2pqpqpq^2pq^2)^2$	2^{13}	1	2 ₂₃	2 ₂₃
2 ₂₄	$p^3qpqp^2qpq^2pqpqpq^2$	2^{13}	1	2 ₂₄	2 ₂₄
2 ₂₅	$(p^2qpq)^3$	$2^{11} \cdot 3$	1	2 ₂₅	2 ₂₅
2 ₂₆	$(p^3q^2pq^2pq^2)^3$	$2^{11} \cdot 3$	1	2 ₂₆	2 ₂₆
2 ₂₇	$(p)^2$	2^{12}	1	2 ₂₇	2 ₂₇
2 ₂₈	$(pq^3)^2$	2^{12}	1	2 ₂₈	2 ₂₈
2 ₂₉	$p^3qp^2qpqpq^2pqp^2q$	2^{12}	1	2 ₂₉	2 ₂₉
2 ₃₀	$p^3qpqpqpqpqpqpqpqpq$	2^{12}	1	2 ₃₀	2 ₃₀
2 ₃₁	$p^3qpq^2pqp^2qpqp^2q$	2^{12}	1	2 ₃₁	2 ₃₁
2 ₃₂	p^2q^3	2^{11}	1	2 ₃₂	2 ₃₂
2 ₃₃	$p^3q^2pqpq^2pqp^2q$	2^{11}	1	2 ₃₃	2 ₃₃
2 ₃₄	$p^3qpqpqpq^2pqpqp^3q^2$	2^{10}	1	2 ₃₄	2 ₃₄
3	$(q)^2$	$2^6 \cdot 3$	3	1	3
4 ₁	$(p^2qpq^4)^3$	$2^{11} \cdot 3$	2 ₂	4 ₁	4 ₁
4 ₂	$(p^3qp^3qpq)^3$	$2^{11} \cdot 3$	2 ₂	4 ₂	4 ₂
4 ₃	$(pq)^4$	2^{12}	2 ₁	4 ₃	4 ₃
4 ₄	$(pqpq^2pq^2)^2$	2^{12}	2 ₁	4 ₄	4 ₄
4 ₅	$(p^3qp^2q)^2$	2^{11}	2 ₁₀	4 ₅	4 ₅
4 ₆	$(p^2q^2pqpqpq^2)^2$	2^{11}	2 ₁₀	4 ₆	4 ₆
4 ₇	$p^3q^2pq^2pq^2pqpq$	2^{11}	2 ₁₀	4 ₇	4 ₇
4 ₈	$p^3qpq^2pqp^2qp^2q^2$	2^{11}	2 ₁₁	4 ₈	4 ₈
4 ₉	$p^3qp^2qp^3qpqp^2q^2$	2^{11}	2 ₁₀	4 ₉	4 ₉
4 ₁₀	$p^3qp^2q^2p^3qpqp^2q$	2^{11}	2 ₁₁	4 ₁₀	4 ₁₀
4 ₁₁	$p^3qpqpq^2p^3q^2p^2q$	2^{11}	2 ₁₁	4 ₁₁	4 ₁₁
4 ₁₂	$p^3qp^2qpqp^2q^2p^3qpq$	2^{11}	2 ₁₁	4 ₁₂	4 ₁₂
4 ₁₃	$(pq^2)^3$	$2^9 \cdot 3$	2 ₁₅	4 ₁₃	4 ₁₃
4 ₁₄	$(p^3q)^3$	$2^9 \cdot 3$	2 ₁₆	4 ₁₄	4 ₁₄
4 ₁₅	$(p^3qpq^2pq)^3$	$2^9 \cdot 3$	2 ₁₅	4 ₁₅	4 ₁₅
4 ₁₆	$(p^3qp^3qpq^2)^3$	$2^9 \cdot 3$	2 ₁₆	4 ₁₆	4 ₁₆
4 ₁₇	$(p^2qpq^2)^2$	2^{10}	2 ₆	4 ₁₇	4 ₁₇
4 ₁₈	$(p^3qpqpq^2)^2$	2^{10}	2 ₁₀	4 ₁₈	4 ₁₈
4 ₁₉	$(p^3qp^2q^2pq)^2$	2^{10}	2 ₁₄	4 ₁₉	4 ₁₉
4 ₂₀	$(p^3qp^2qpqpq)^2$	2^{10}	2 ₈	4 ₂₀	4 ₂₀
4 ₂₁	$p^2qpqp^2qpq^2$	2^{10}	2 ₈	4 ₂₁	4 ₂₁
4 ₂₂	$p^3qp^2qpqpq^2pq^2$	2^{10}	2 ₁₄	4 ₂₂	4 ₂₂
4 ₂₃	$p^3qpqpqpq^2qpq^4$	2^{10}	2 ₁₀	4 ₂₃	4 ₂₃
4 ₂₄	$p^3qpqpqpq^2pqp^2q$	2^{10}	2 ₁₁	4 ₂₄	4 ₂₄
4 ₂₅	$p^3qp^2q^2p^3qpqpq$	2^{10}	2 ₁₁	4 ₂₅	4 ₂₅
4 ₂₆	$p^3qpq^2pqp^3q^2pq^2$	2^{10}	2 ₁₀	4 ₂₆	4 ₂₆
4 ₂₇	$p^2qp^2q^2pq^2pq^2pqp^2q^2$	2^{10}	2 ₂	4 ₂₇	4 ₂₇
4 ₂₈	$p^3q^2pqp^2q^2p^2q^2pqpq^2$	2^{10}	2 ₆	4 ₂₈	4 ₂₈
4 ₂₉	$(p^3q^2pqpq)^2$	2^9	2 ₁₉	4 ₂₉	4 ₂₉
4 ₃₀	$(p^2qpq^2p^2q^2)^2$	2^9	2 ₁₉	4 ₃₀	4 ₃₀
4 ₃₁	$p^3q^2pq^2p^2q$	2^9	2 ₁₆	4 ₃₁	4 ₃₁
4 ₃₂	p^2qpqpq^3	2^9	2 ₁₆	4 ₃₂	4 ₃₂
4 ₃₃	$p^3qpqpqp^2q^2$	2^9	2 ₁₅	4 ₃₃	4 ₃₃
4 ₃₄	$p^2qpq^2pqp^2q^2$	2^9	2 ₁₁	4 ₃₄	4 ₃₄
4 ₃₅	$p^3q^2p^2qpqpq$	2^9	2 ₁₆	4 ₃₅	4 ₃₅

Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle$, with subscripts dropped (continued)

Class	Representative	Centralizer	2P	3P	5P
436	$p^2 q^2 p q p q p q^2$	2^9	2 ₁₄	4 ₃₆	4 ₃₆
437	$p^3 q p^2 q^2 p q p^2 q$	2^9	2 ₁₆	4 ₃₇	4 ₃₇
438	$p^3 q p^2 q^2 p q^2 p^2 q$	2^9	2 ₁₃	4 ₃₈	4 ₃₈
439	$p^3 q^2 p q p q p q p q$	2^9	2 ₁₆	4 ₃₉	4 ₃₉
440	$p^3 q p^2 q p q^2 p q p q^2$	2^9	2 ₁₅	4 ₄₀	4 ₄₀
441	$p^3 q p^2 q p q p^2 q p^2 q$	2^9	2 ₁₄	4 ₄₁	4 ₄₁
442	$p^3 q^2 p q p q p^2 q p q p q$	2^9	2 ₁₁	4 ₄₂	4 ₄₂
443	$p^3 q p^2 q^2 p q^2 p q^2 p q$	2^9	2 ₁₃	4 ₄₃	4 ₄₃
444	$p^3 q^2 p q p^2 q^2 p q p q^2 p q$	2^9	2 ₁₆	4 ₄₄	4 ₄₄
445	$p^3 q p^2 q^2 p^2 q p q p q p^2 q$	2^9	2 ₁₀	4 ₄₅	4 ₄₅
446	$p^3 q p q p q^2$	2^8	2 ₁₆	4 ₄₆	4 ₄₆
447	$p^3 q^2 p q p q^2$	2^8	2 ₂₀	4 ₄₇	4 ₄₇
448	$p^3 q p q p^3 q^2$	2^8	2 ₁₉	4 ₄₈	4 ₄₈
449	$p^2 q p q p q^2 p^2 q^2$	2^8	2 ₂₀	4 ₄₉	4 ₄₉
450	$p^2 q p q p q p^2 q p q^2$	2^8	2 ₂₀	4 ₅₀	4 ₅₀
451	$p^3 q^2 p q p q p q^2 p q^2$	2^8	2 ₂₃	4 ₅₁	4 ₅₁
452	$p^3 q p q p^3 q p q p q p q$	2^8	2 ₁₆	4 ₅₂	4 ₅₂
453	$p^2 q p^2 q^2 p q p q^2 q p q$	2^8	2 ₁₉	4 ₅₃	4 ₅₃
454	$p^2 q p q p^2 q^2 p q p q p q^2$	2^8	2 ₂₃	4 ₅₄	4 ₅₄
455	p	2^7	2 ₂₇	4 ₅₅	4 ₅₅
456	$p q^3$	2^7	2 ₂₈	4 ₅₆	4 ₅₆
457	$p^2 q p q p q p q^2$	2^7	2 ₂₇	4 ₅₇	4 ₅₇
458	$p^2 q^2 p^2 q^2 p q p q$	2^7	2 ₂₀	4 ₅₈	4 ₅₈
459	$p^2 q p q p q p q^3$	2^7	2 ₂₈	4 ₅₉	4 ₅₉
5	$(p^2 q)^2$	$2^3 \cdot 5$	5	5	1
6 ₁	$(p q^2)^2$	$2^6 \cdot 3$	3	2 ₁₅	6 ₁
6 ₂	$(p^3 q)^2$	$2^6 \cdot 3$	3	2 ₁₆	6 ₂
6 ₃	$p^3 q p q^3$	$2^6 \cdot 3$	3	2 ₂	6 ₃
6 ₄	$p q p q p q p q^2$	$2^6 \cdot 3$	3	2 ₁₇	6 ₄
6 ₅	$p^3 q^2 p^2 q p q^2$	$2^6 \cdot 3$	3	2 ₃	6 ₅
6 ₆	$p^3 q^2 p^2 q^4 p q$	$2^6 \cdot 3$	3	2 ₁	6 ₆
6 ₇	$p^3 q p q^2 p q p q^4$	$2^6 \cdot 3$	3	2 ₁₈	6 ₇
6 ₈	$p^3 q$	$2^5 \cdot 3$	3	2 ₄	6 ₈
6 ₉	$p^3 q p q$	$2^5 \cdot 3$	3	2 ₁₂	6 ₁₀
6 ₁₀	$p^3 q p^2 q p q$	$2^5 \cdot 3$	3	2 ₁₂	6 ₉
6 ₁₁	$p^3 q p q p^2 q$	$2^5 \cdot 3$	3	2 ₁₃	6 ₁₉
6 ₁₂	$p^3 q p q^4$	$2^5 \cdot 3$	3	2 ₇	6 ₁₂
6 ₁₃	$p^3 q^4 p q$	$2^5 \cdot 3$	3	2 ₆	6 ₁₃
6 ₁₄	$p q^3 p q^5$	$2^5 \cdot 3$	3	2 ₂₁	6 ₁₄
6 ₁₅	$p^3 q^2 p^2 q p q$	$2^5 \cdot 3$	3	2 ₅	6 ₁₅
6 ₁₆	$p^3 q p q^2 p q p q$	$2^5 \cdot 3$	3	2 ₂₂	6 ₁₆
6 ₁₇	$p^3 q p q p^2 q^4$	$2^5 \cdot 3$	3	2 ₉	6 ₁₇
6 ₁₈	$p^3 q p q p q^3 p q$	$2^5 \cdot 3$	3	2 ₈	6 ₁₈
6 ₁₉	$p^3 q^2 p^2 q p q p^2 q$	$2^5 \cdot 3$	3	2 ₁₃	6 ₁₁
6 ₂₀	$p^2 q p q$	$2^4 \cdot 3$	3	2 ₂₅	6 ₂₀
6 ₂₁	$p^3 q^2 p q^2 p q^2$	$2^4 \cdot 3$	3	2 ₂₆	6 ₂₁
8 ₁	$(p q)^2$	2^7	4 ₃	8 ₁	8 ₁
8 ₂	$p^3 q p^2 q$	2^7	4 ₅	8 ₂	8 ₂
8 ₃	$p^2 q^2 p q p q$	2^7	4 ₅	8 ₃	8 ₃
8 ₄	$p q p q^2 p q^2$	2^7	4 ₄	8 ₄	8 ₄
8 ₅	$p^2 q p^2 q p q^2$	2^7	4 ₄	8 ₅	8 ₅
8 ₆	$p^2 q^2 p q p q^3$	2^7	4 ₅	8 ₆	8 ₆
8 ₇	$p^3 q p q p q^2 p^2 q^2$	2^7	4 ₅	8 ₇	8 ₇
8 ₈	$p^2 q p q^2$	2^6	4 ₁₇	8 ₈	8 ₈
8 ₉	$p^3 q p q^2$	2^6	4 ₃	8 ₉	8 ₉
8 ₁₀	$p^3 q p q p q^2$	2^6	4 ₁₈	8 ₁₆	8 ₁₀
8 ₁₁	$p^2 q p q p q p q$	2^6	4 ₁₇	8 ₁₁	8 ₁₁
8 ₁₂	$p^3 q p^2 q^2 p q$	2^6	4 ₁₉	8 ₁₂	8 ₁₂
8 ₁₃	$p^3 q p^2 q p q p q$	2^6	4 ₂₀	8 ₁₃	8 ₁₃
8 ₁₄	$p^3 q p^2 q^2 p q^2$	2^6	4 ₁₉	8 ₁₄	8 ₁₄
8 ₁₅	$p^2 q^2 p q p q p q^2$	2^6	4 ₆	8 ₁₅	8 ₁₅
8 ₁₆	$p^3 q p q p q^3$	2^6	4 ₁₈	8 ₁₀	8 ₁₆
8 ₁₇	$p^3 q p q^2 p^2 q p q$	2^6	4 ₂₀	8 ₁₇	8 ₁₇
8 ₁₈	$p^3 q^2 p q p q$	2^5	4 ₂₉	8 ₁₈	8 ₁₈
8 ₁₉	$p^2 q p q^2 p^2 q^2$	2^5	4 ₃₀	8 ₁₉	8 ₁₉
10 ₁	$p^2 q$	$2^3 \cdot 5$	5	10 ₁	2 ₂
10 ₂	$p^2 q^2$	$2^3 \cdot 5$	5	10 ₃	2 ₄
10 ₃	$p q p q^3$	$2^3 \cdot 5$	5	10 ₂	2 ₄
10 ₄	$p^2 q p q^2 p q^2$	$2^3 \cdot 5$	5	10 ₄	2 ₃
10 ₅	$p^2 q^2 p q^2 p q^2$	$2^3 \cdot 5$	5	10 ₅	2 ₁
10 ₆	$p^3 q p^3 q p^2 q$	$2^3 \cdot 5$	5	10 ₇	2 ₅
10 ₇	$p^3 q^2 p q p q p q$	$2^3 \cdot 5$	5	10 ₆	2 ₅

Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle$, with subscripts dropped (continued)

<i>Class</i>	<i>Representative</i>	<i>Centralizer</i>	2P	3P	5P
12 ₁	pq^2	$2^4 \cdot 3$	6 ₁	4 ₁₃	12 ₁
12 ₂	p^3q	$2^4 \cdot 3$	6 ₂	4 ₁₄	12 ₂
12 ₃	p^2qpq^4	$2^4 \cdot 3$	6 ₃	4 ₁	12 ₃
12 ₄	p^3qpq^2pq	$2^4 \cdot 3$	6 ₁	4 ₁₅	12 ₄
12 ₅	p^3qp^3qpq	$2^4 \cdot 3$	6 ₃	4 ₂	12 ₅
12 ₆	$p^3qp^3qpq^2$	$2^4 \cdot 3$	6 ₂	4 ₁₆	12 ₆
16 ₁	pq	2^5	8 ₁	16 ₁	16 ₂
16 ₂	p^3q^2	2^5	8 ₁	16 ₂	16 ₁

A.5. Conjugacy classes of $H(\text{Fi}_{23}) = H = \langle x_0, y_0, h_0 \rangle$, with subscripts dropped

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
1	1	$2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	1	1	1	1	1
2 ₁	$(xy^2)^7$	$2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	2 ₁	2 ₁	2 ₁	2 ₁	2 ₁
2 ₂	$(y)^7$	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	1	2 ₂	2 ₂	2 ₂	2 ₂	2 ₂
2 ₃	$(xyhy)^{11}$	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	1	2 ₃	2 ₃	2 ₃	2 ₃	2 ₃
2 ₄	x	$2^{18} \cdot 3^4 \cdot 5$	1	2 ₄	2 ₄	2 ₄	2 ₄	2 ₄
2 ₅	$(yh)^{10}$	$2^{18} \cdot 3^4 \cdot 5$	1	2 ₅	2 ₅	2 ₅	2 ₅	2 ₅
2 ₆	$(xh)^6$	$2^{16} \cdot 3^3$	1	2 ₆	2 ₆	2 ₆	2 ₆	2 ₆
3 ₁	$(xyxh)^{10}$	$2^9 \cdot 3^7 \cdot 5 \cdot 7$	3 ₁	1	3 ₁	3 ₁	3 ₁	3 ₁
3 ₂	h	$2^8 \cdot 3^9$	3 ₂	1	3 ₂	3 ₂	3 ₂	3 ₂
3 ₃	$(xyxy^3)^4$	$2^7 \cdot 3^7$	3 ₃	1	3 ₃	3 ₃	3 ₃	3 ₃
3 ₄	$(xyhyh)^6$	$2^4 \cdot 3^7$	3 ₄	1	3 ₄	3 ₄	3 ₄	3 ₄
4 ₁	$(xy^2xh)^3$	$2^{12} \cdot 3^3$	2 ₄	4 ₁	4 ₁	4 ₁	4 ₁	4 ₁
4 ₂	$(yh)^5$	$2^{11} \cdot 3^2 \cdot 5$	2 ₅	4 ₂	4 ₂	4 ₂	4 ₂	4 ₂
4 ₃	$(xhy)^5$	$2^{11} \cdot 3^2 \cdot 5$	2 ₅	4 ₃	4 ₃	4 ₃	4 ₃	4 ₃
4 ₄	$(y^2h)^4$	$2^{12} \cdot 3$	2 ₄	4 ₄	4 ₄	4 ₄	4 ₄	4 ₄
4 ₅	$(xh)^3$	$2^{10} \cdot 3^2$	2 ₆	4 ₅	4 ₅	4 ₅	4 ₅	4 ₅
4 ₆	$(xyh)^2$	$2^{10} \cdot 3^2$	2 ₆	4 ₆	4 ₆	4 ₆	4 ₆	4 ₆
4 ₇	$(xh^2y^3)^3$	$2^{10} \cdot 3$	2 ₅	4 ₇	4 ₇	4 ₇	4 ₇	4 ₇
5	$(yh)^4$	$2^4 \cdot 3 \cdot 5^2$	5	5	1	5	5	5
6 ₁	$(y^3h^2)^5$	$2^9 \cdot 3^7 \cdot 5 \cdot 7$	3 ₁	2 ₁	6 ₁	6 ₁	6 ₁	6 ₁
6 ₂	$(xy^2h^2yhy)^3$	$2^8 \cdot 3^9$	3 ₂	2 ₁	6 ₂	6 ₂	6 ₂	6 ₂
6 ₃	$(xyxh)^5$	$2^8 \cdot 3^5 \cdot 5$	3 ₁	2 ₂	6 ₃	6 ₃	6 ₃	6 ₃
6 ₄	$(xhy^3)^5$	$2^8 \cdot 3^5 \cdot 5$	3 ₁	2 ₃	6 ₄	6 ₄	6 ₄	6 ₄
6 ₅	$xyh^2yh^2xh^2xh$	$2^7 \cdot 3^7$	3 ₃	2 ₁	6 ₅	6 ₅	6 ₅	6 ₅
6 ₆	$(xyxyh)^3$	$2^8 \cdot 3^6$	3 ₂	2 ₃	6 ₆	6 ₆	6 ₆	6 ₆
6 ₇	$(xyxhy)^3$	$2^8 \cdot 3^6$	3 ₂	2 ₂	6 ₇	6 ₇	6 ₇	6 ₇
6 ₈	$(xyhyh)^3$	$2^4 \cdot 3^7$	3 ₄	2 ₁	6 ₈	6 ₈	6 ₈	6 ₈
6 ₉	$(xy^2xh)^2$	$2^8 \cdot 3^4$	3 ₂	2 ₄	6 ₉	6 ₉	6 ₉	6 ₉
6 ₁₀	$(xy^2xyh)^3$	$2^8 \cdot 3^4$	3 ₂	2 ₅	6 ₁₀	6 ₁₀	6 ₁₀	6 ₁₀
6 ₁₁	$(yhyh^2)^4$	$2^9 \cdot 3^3$	3 ₁	2 ₄	6 ₁₁	6 ₁₁	6 ₁₁	6 ₁₁
6 ₁₂	$(xh^2y^3)^2$	$2^9 \cdot 3^3$	3 ₁	2 ₅	6 ₁₂	6 ₁₂	6 ₁₂	6 ₁₂
6 ₁₃	y^7h	$2^5 \cdot 3^5$	3 ₃	2 ₂	6 ₁₃	6 ₁₃	6 ₁₃	6 ₁₃
6 ₁₄	$xyxyxhyxhy^2$	$2^5 \cdot 3^5$	3 ₃	2 ₃	6 ₁₄	6 ₁₄	6 ₁₄	6 ₁₄
6 ₁₅	$(xh)^2$	$2^8 \cdot 3^3$	3 ₂	2 ₆	6 ₁₅	6 ₁₅	6 ₁₅	6 ₁₅
6 ₁₆	$(xhxhy^2h)^2$	$2^8 \cdot 3^3$	3 ₂	2 ₆	6 ₁₆	6 ₁₆	6 ₁₆	6 ₁₆
6 ₁₇	$xyhaxhaxh$	$2^7 \cdot 3^3$	3 ₃	2 ₆	6 ₁₇	6 ₁₇	6 ₁₇	6 ₁₇
6 ₁₈	$(xy^2xhyhy)^2$	$2^7 \cdot 3^3$	3 ₃	2 ₆	6 ₁₈	6 ₁₈	6 ₁₈	6 ₁₈
6 ₁₉	$xyxy^2xyxyhy^2$	$2^7 \cdot 3^3$	3 ₁	2 ₆	6 ₁₉	6 ₁₉	6 ₁₉	6 ₁₉
6 ₂₀	$(xyxy^3)^2$	$2^6 \cdot 3^3$	3 ₃	2 ₅	6 ₂₀	6 ₂₀	6 ₂₀	6 ₂₀
6 ₂₁	xh^2yh^2y	$2^6 \cdot 3^3$	3 ₃	2 ₄	6 ₂₁	6 ₂₁	6 ₂₁	6 ₂₁
6 ₂₂	$xyxhy^2xyh$	$2^5 \cdot 3^3$	3 ₃	2 ₆	6 ₂₂	6 ₂₂	6 ₂₂	6 ₂₂
6 ₂₃	$xyxh^2yhyh$	$2^5 \cdot 3^3$	3 ₃	2 ₆	6 ₂₃	6 ₂₃	6 ₂₃	6 ₂₃
6 ₂₄	$(xyxyhyh)^2$	$2^4 \cdot 3^3$	3 ₄	2 ₆	6 ₂₄	6 ₂₄	6 ₂₄	6 ₂₄
6 ₂₅	xyh^2y^2xh	$2^4 \cdot 3^3$	3 ₄	2 ₆	6 ₂₅	6 ₂₅	6 ₂₅	6 ₂₅
7	$(y)^2$	$2^2 \cdot 3 \cdot 7$	7	7	7	1	7	7
8 ₁	$(yhyh^2)^3$	$2^7 \cdot 3$	4 ₁	8 ₁	8 ₁	8 ₁	8 ₁	8 ₁
8 ₂	$(xyhy^2h)^3$	$2^7 \cdot 3$	4 ₁	8 ₂	8 ₂	8 ₂	8 ₂	8 ₂
8 ₃	$(y^2h)^2$	2^7	4 ₄	8 ₃	8 ₃	8 ₃	8 ₃	8 ₃
8 ₄	xyh	2^6	4 ₆	8 ₄	8 ₄	8 ₄	8 ₄	8 ₄
8 ₅	xyh^2yh	2^6	4 ₆	8 ₅	8 ₅	8 ₅	8 ₅	8 ₅
9 ₁	$(xyxyh)^2$	$2^3 \cdot 3^4$	9 ₁	3 ₂	9 ₁	9 ₁	9 ₁	9 ₁
9 ₂	$(xyhy^3)^2$	$2^2 \cdot 3^4$	9 ₂	3 ₂	9 ₂	9 ₂	9 ₂	9 ₂
9 ₃	$(xyhyh)^2$	$2 \cdot 3^3$	9 ₃	3 ₄	9 ₃	9 ₃	9 ₃	9 ₃
10 ₁	$(y^3h^2)^3$	$2^4 \cdot 3 \cdot 5^2$	5	10 ₁	2 ₁	10 ₁	10 ₁	10 ₁
10 ₂	yh^2	$2^3 \cdot 3 \cdot 5$	5	10 ₂	2 ₂	10 ₂	10 ₂	10 ₂
10 ₃	$(xhy^3)^3$	$2^3 \cdot 3 \cdot 5$	5	10 ₃	2 ₃	10 ₃	10 ₃	10 ₃
10 ₄	$(yh)^2$	$2^4 \cdot 5$	5	10 ₄	2 ₅	10 ₄	10 ₄	10 ₄
10 ₅	xh^2y	$2^4 \cdot 5$	5	10 ₅	2 ₄	10 ₅	10 ₅	10 ₅
11 ₁	$(xy^3)^2$	$2^2 \cdot 11$	11 ₂	11 ₁	11 ₁	11 ₂	1	11 ₂
11 ₂	$(xy^3)^4$	$2^2 \cdot 11$	11 ₁	11 ₂	11 ₂	11 ₁	1	11 ₁
12 ₁	$(yhyh^2)^2$	$2^7 \cdot 3^2$	6 ₁₁	4 ₁	12 ₁	12 ₁	12 ₁	12 ₁
12 ₂	$(xyhy^2h)^2$	$2^7 \cdot 3^2$	6 ₁₁	4 ₁	12 ₂	12 ₂	12 ₂	12 ₂
12 ₃	xy^2xh	$2^5 \cdot 3^3$	6 ₉	4 ₁	12 ₃	12 ₃	12 ₃	12 ₃
12 ₄	xy^2xy^3h	$2^6 \cdot 3^2$	6 ₁₂	4 ₃	12 ₄	12 ₄	12 ₄	12 ₄
12 ₅	$xyxy^3xh^2$	$2^6 \cdot 3^2$	6 ₁₂	4 ₂	12 ₅	12 ₅	12 ₅	12 ₅
12 ₆	$xy^2xhxhyhy$	$2^6 \cdot 3^2$	6 ₁₁	4 ₁	12 ₆	12 ₆	12 ₆	12 ₆
12 ₇	xh	$2^5 \cdot 3^2$	6 ₁₅	4 ₅	12 ₇	12 ₇	12 ₇	12 ₇
12 ₈	$xyxh^2y$	$2^5 \cdot 3^2$	6 ₁₅	4 ₆	12 ₈	12 ₈	12 ₈	12 ₈
12 ₉	$xhxhy^2h$	$2^5 \cdot 3^2$	6 ₁₆	4 ₆	12 ₉	12 ₉	12 ₉	12 ₉
12 ₁₀	xy^2xhyhy	$2^5 \cdot 3^2$	6 ₁₈	4 ₅	12 ₁₀	12 ₁₀	12 ₁₀	12 ₁₀
12 ₁₁	$xhxh^2y^4$	$2^5 \cdot 3^2$	6 ₁₈	4 ₆	12 ₁₁	12 ₁₁	12 ₁₁	12 ₁₁

Conjugacy classes of $H = \langle x_0, y_0, h_0 \rangle$, with subscripts dropped (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
12 ₁₂	xhy^4hyh	$2^5 \cdot 3^2$	6 ₁₆	4 ₅	12 ₁₂	12 ₁₂	12 ₁₂	12 ₁₂
12 ₁₃	$xyxy^3$	$2^4 \cdot 3^2$	6 ₂₀	4 ₂	12 ₁₃	12 ₁₃	12 ₁₃	12 ₁₃
12 ₁₄	$xyxhy^3h$	$2^4 \cdot 3^2$	6 ₂₀	4 ₃	12 ₁₄	12 ₁₄	12 ₁₄	12 ₁₄
12 ₁₅	xh^2y^3	$2^5 \cdot 3$	6 ₁₂	4 ₇	12 ₁₅	12 ₁₅	12 ₁₅	12 ₁₅
12 ₁₆	xy^3hy^2h	$2^5 \cdot 3$	6 ₉	4 ₄	12 ₁₆	12 ₁₆	12 ₁₆	12 ₁₆
12 ₁₇	$xyxyhyh$	$2^3 \cdot 3^2$	6 ₂₄	4 ₆	12 ₁₇	12 ₁₇	12 ₁₇	12 ₁₇
12 ₁₈	xy^2h^2xh	$2^3 \cdot 3^2$	6 ₂₄	4 ₅	12 ₁₈	12 ₁₈	12 ₁₈	12 ₁₈
13 ₁	$(xy^3h)^2$	$2 \cdot 13$	13 ₂	13 ₁	13 ₂	13 ₂	13 ₂	1
13 ₂	$(xy^3h)^4$	$2 \cdot 13$	13 ₁	13 ₂	13 ₁	13 ₁	13 ₁	1
14 ₁	xy^2	$2^2 \cdot 3 \cdot 7$	7	14 ₁	14 ₁	2 ₁	14 ₁	14 ₁
14 ₂	y	$2^2 \cdot 7$	7	14 ₂	14 ₂	2 ₂	14 ₂	14 ₂
14 ₃	xh^2y^2	$2^2 \cdot 7$	7	14 ₃	14 ₃	2 ₃	14 ₃	14 ₃
15	$(xyxh)^2$	$2^2 \cdot 3 \cdot 5$	15	5	3 ₁	15	15	15
16 ₁	y^2h	2^5	8 ₃	16 ₁	16 ₂	16 ₂	16 ₁	16 ₂
16 ₂	$xyxyhy$	2^5	8 ₃	16 ₂	16 ₁	16 ₁	16 ₂	16 ₁
18 ₁	xhy^2hy^2h	$2^3 \cdot 3^4$	9 ₁	6 ₂	18 ₁	18 ₁	18 ₁	18 ₁
18 ₂	xy^2h^2yhy	$2^2 \cdot 3^4$	9 ₂	6 ₂	18 ₂	18 ₂	18 ₂	18 ₂
18 ₃	$xyxyhy$	$2^3 \cdot 3^3$	9 ₁	6 ₆	18 ₅	18 ₃	18 ₅	18 ₃
18 ₄	$xyxhy$	$2^3 \cdot 3^3$	9 ₁	6 ₇	18 ₆	18 ₄	18 ₆	18 ₄
18 ₅	xy^2xhy	$2^3 \cdot 3^3$	9 ₁	6 ₆	18 ₃	18 ₅	18 ₃	18 ₅
18 ₆	xhy^3h^2	$2^3 \cdot 3^3$	9 ₁	6 ₇	18 ₄	18 ₆	18 ₄	18 ₆
18 ₇	$xyhy^3$	$2^2 \cdot 3^3$	9 ₂	6 ₆	18 ₇	18 ₇	18 ₇	18 ₇
18 ₈	$xyxh^2xh$	$2^2 \cdot 3^3$	9 ₂	6 ₇	18 ₈	18 ₈	18 ₈	18 ₈
18 ₉	xy^2xyh	$2^3 \cdot 3^2$	9 ₁	6 ₁₀	18 ₉	18 ₉	18 ₉	18 ₉
18 ₁₀	xh^2yhy	$2^3 \cdot 3^2$	9 ₁	6 ₉	18 ₁₀	18 ₁₀	18 ₁₀	18 ₁₀
18 ₁₁	$xyhyh$	$2 \cdot 3^3$	9 ₃	6 ₈	18 ₁₁	18 ₁₁	18 ₁₁	18 ₁₁
20 ₁	yh	$2^3 \cdot 5$	10 ₄	20 ₁	4 ₂	20 ₁	20 ₁	20 ₁
20 ₂	xhy	$2^3 \cdot 5$	10 ₄	20 ₂	4 ₃	20 ₂	20 ₂	20 ₂
21	$xhyhy$	$2 \cdot 3 \cdot 7$	21	7	21	3 ₁	21	21
22 ₁	xy^3	$2^2 \cdot 11$	11 ₁	22 ₁	22 ₁	22 ₃	2 ₁	22 ₃
22 ₂	$xyhy$	$2^2 \cdot 11$	11 ₁	22 ₂	22 ₂	22 ₅	2 ₃	22 ₅
22 ₃	$xyxh^2$	$2^2 \cdot 11$	11 ₂	22 ₃	22 ₃	22 ₁	2 ₁	22 ₁
22 ₄	$xyxhy^2$	$2^2 \cdot 11$	11 ₂	22 ₄	22 ₄	22 ₆	2 ₂	22 ₆
22 ₅	$xyhxh^2$	$2^2 \cdot 11$	11 ₂	22 ₅	22 ₅	22 ₂	2 ₃	22 ₂
22 ₆	xy^2xy^2h	$2^2 \cdot 11$	11 ₁	22 ₆	22 ₆	22 ₄	2 ₂	22 ₄
24 ₁	$yhhyh^2$	$2^4 \cdot 3$	12 ₁	8 ₁	24 ₁	24 ₁	24 ₁	24 ₁
24 ₂	$xyhy^2h$	$2^4 \cdot 3$	12 ₂	8 ₂	24 ₂	24 ₂	24 ₂	24 ₂
26 ₁	xy^3h	$2 \cdot 13$	13 ₁	26 ₁	26 ₂	26 ₂	26 ₂	2 ₁
26 ₂	xhy^2h	$2 \cdot 13$	13 ₂	26 ₂	26 ₁	26 ₁	26 ₁	2 ₁
30 ₁	$xyxh$	$2^2 \cdot 3 \cdot 5$	15	10 ₂	6 ₃	30 ₁	30 ₁	30 ₁
30 ₂	xhy^3	$2^2 \cdot 3 \cdot 5$	15	10 ₃	6 ₄	30 ₂	30 ₂	30 ₂
30 ₃	y^3h^2	$2^2 \cdot 3 \cdot 5$	15	10 ₁	6 ₁	30 ₃	30 ₃	30 ₃
42	xhy^2hy	$2 \cdot 3 \cdot 7$	21	14 ₁	42	6 ₁	42	42

Conjugacy classes of $\mathfrak{G} = \langle x, \eta, h, \epsilon \rangle$ (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P	17P	23P
18 ₇	$x\eta h y h$	$2 \cdot 3^3$	9 ₅	6 ₁₀	18 ₇	18 ₇	18 ₇	18 ₇	18 ₇	18 ₇
18 ₈	$y h^2 \epsilon h^2$	$2 \cdot 3^3$	9 ₄	6 ₈	18 ₈	18 ₈	18 ₈	18 ₈	18 ₈	18 ₈
20 ₁	$x h y$	$2^3 \cdot 3 \cdot 5$	10 ₂	20 ₁	4 ₁	20 ₁	20 ₁	20 ₁	20 ₁	20 ₁
20 ₂	$y h$	$2^3 \cdot 5$	10 ₂	20 ₂	4 ₃	20 ₂	20 ₂	20 ₂	20 ₂	20 ₂
21	$y h \epsilon$	$2 \cdot 3 \cdot 7$	21	7	21	3 ₁	21	21	21	21
22 ₁	$x y^3$	$2^2 \cdot 11$	11	22 ₁	22 ₁	22 ₃	2 ₁	22 ₃	22 ₃	22 ₁
22 ₂	$x y h y$	$2^2 \cdot 11$	11	22 ₂	22 ₂	22 ₂	2 ₂	22 ₂	22 ₂	22 ₂
22 ₃	$x \epsilon y^2$	$2^2 \cdot 11$	11	22 ₃	22 ₃	22 ₁	2 ₁	22 ₁	22 ₁	22 ₃
23 ₁	$x y \epsilon h^2$	23	23 ₁	23 ₁	23 ₂	23 ₂	23 ₂	23 ₁	23 ₂	1
23 ₂	$y h^2 y \epsilon$	23	23 ₂	23 ₂	23 ₁	23 ₁	23 ₁	23 ₂	23 ₁	1
24 ₁	$x h \epsilon h$	$2^4 \cdot 3$	12 ₂	8 ₃	24 ₁	24 ₁	24 ₁	24 ₁	24 ₁	24 ₁
24 ₂	$x y h y \epsilon$	$2^4 \cdot 3$	12 ₆	8 ₂	24 ₂	24 ₂	24 ₂	24 ₂	24 ₂	24 ₂
24 ₃	$x y h y^2 h$	$2^4 \cdot 3$	12 ₇	8 ₁	24 ₃	24 ₃	24 ₃	24 ₃	24 ₃	24 ₃
26 ₁	$x y^3 h$	$2 \cdot 13$	13 ₁	26 ₁	26 ₂	26 ₂	2 ₁	26 ₁	26 ₁	26 ₁
26 ₂	$x y^2 \epsilon h$	$2 \cdot 13$	13 ₂	26 ₂	26 ₁	26 ₁	2 ₁	26 ₂	26 ₂	26 ₂
27	$x y h^2 \epsilon$	3^3	27	9 ₁	27	27	27	27	27	27
28	$x h y h \epsilon$	$2^2 \cdot 7$	14 ₂	28	28	4 ₁	28	28	28	28
30 ₁	$x h y^3$	$2^3 \cdot 3 \cdot 5$	15 ₁	10 ₂	6 ₃	30 ₁	30 ₁	30 ₁	30 ₁	30 ₁
30 ₂	$x y x h$	$2^2 \cdot 3 \cdot 5$	15 ₁	10 ₁	6 ₁	30 ₂	30 ₂	30 ₂	30 ₂	30 ₂
30 ₃	$x \epsilon h y$	$2 \cdot 3 \cdot 5$	15 ₂	10 ₃	6 ₇	30 ₃	30 ₃	30 ₃	30 ₃	30 ₃
35	$y^2 h \epsilon$	$5 \cdot 7$	35	35	7	5	35	35	35	35
36 ₁	$x y h \epsilon y$	$2^2 \cdot 3^2$	18 ₅	12 ₅	36 ₁	36 ₁	36 ₁	36 ₁	36 ₁	36 ₁
36 ₂	$x y^2 \epsilon y h$	$2^2 \cdot 3^2$	18 ₆	12 ₄	36 ₂	36 ₂	36 ₂	36 ₂	36 ₂	36 ₂
39 ₁	$x h y h \epsilon$	$3 \cdot 13$	39 ₂	13 ₂	39 ₂	39 ₂	39 ₂	3 ₁	39 ₁	39 ₁
39 ₂	$x h^2 x \epsilon y$	$3 \cdot 13$	39 ₁	13 ₁	39 ₁	39 ₁	39 ₁	3 ₁	39 ₂	39 ₂
42	$y \epsilon y \epsilon h$	$2 \cdot 3 \cdot 7$	21	14 ₁	42	6 ₁	42	42	42	42
60	$x \epsilon y h^2$	$2^2 \cdot 3 \cdot 5$	30 ₁	20 ₁	12 ₁	60	60	60	60	60

APPENDIX B. CHARACTER TABLES OF LOCAL SUBGROUPS OF Fi_{23} AND 2Fi_{22} B.1. Character table of $E(\text{Fi}_{23}) = E = \langle x_1, y_1, e_1 \rangle$

2	18	18	18	18	14	14	7	11	12	12	11	9	9	8	3	7	7	7	6	6	5	5	3	3
3	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	1	1
5	1	1	1	1	
7	1	1	1	1	
11	1	1	1	1	
23	1	1	1	1	
	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
2P	1a	1a	1a	1a	1a	1a	3a	2b	2c	2c	2b	2e	2e	2d	5a	3a	3a	3a	3a	3a	3a	3a	7a	7b
3P	1a	2a	2b	2c	2d	2e	1a	4a	4b	4c	4d	4e	4f	4g	5a	2c	2c	2c	2b	2c	2d	2e	7b	7a
5P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	1a	6a	6b	6c	6d	6e	6f	6g	7b	7a
7P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	1a	1a
11P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
23P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	22	22	22	22	6	6	4	6	6	6	6	2	2	2	2	4	4	4	4	4	4	4	4	1
X.3	45	45	45	45	-3	-3	-3	-3	-3	-3	1	1	1	
X.4	45	45	45	45	-3	-3	-3	-3	-3	-3	1	1	1	
X.5	230	230	230	230	22	22	5	22	22	22	22	2	2	2	5	5	5	5	5	5	1	1	-1
X.6	231	231	231	231	7	7	6	7	7	7	7	7	-1	-1	1	6	6	6	6	6	6	-2	-2
X.7	231	231	231	231	7	7	-3	7	7	7	7	-1	-1	-1	1	-3	-3	-3	-3	-3	-3	1	1
X.8	231	231	231	231	7	7	-3	7	7	7	7	-1	-1	-1	1	-3	-3	-3	-3	-3	-3	1	1
X.9	253	253	253	253	13	13	1	13	13	13	13	1	1	1	-2	1	1	1	1	1	1	1	1	1
X.10	253	-99	29	-3	29	-3	10	-15	-3	5	1	5	-3	1	3	-6	
X.11	506	154	26	-6	42	10	11	14	-6	2	-2	6	-2	2	1	9	3	1	-1	-3	3	1	2	2
X.12	770	770	770	770	-14	-14	5	-14	-14	-14	-14	-2	-2	-2	5	5	5	5	5	1	1	
X.13	770	770	770	770	-14	-14	5	-14	-14	-14	-14	-2	-2	-2	5	5	5	5	5	1	1	
X.14	896	896	896	896	-4	1	-4	-4	-4	-4	-4	
X.15	896	896	896	896	-4	1	-4	-4	-4	-4	-4	
X.16	990	990	990	990	-18	-18	-18	-18	-18	-18	2	2	2	
X.17	990	990	990	990	-18	-18	-18	-18	-18	-18	2	2	2	
X.18	1035	1035	1035	1035	27	27	27	27	27	27	-1	-1	-1	
X.19	1288	-56	-56	8	56	-8	10	8	-8	4	4	-4	3	-10	2	-2	-2	2	2	-2	-2	
X.20	1518	-594	174	-18	62	-2	15	-34	-2	14	-2	2	2	2	3	-15	-9	9	3	-3	-1	-1	-1	
X.21	2024	2024	2024	2024	8	8	-1	8	8	8	8	-1	-1	-1	-1	-1	-1	-1	-1	1	
X.22	2530	-990	290	-30	-46	18	10	18	18	2	-14	-2	-2	-2	-6	
X.23	2530	-990	290	-30	-46	18	10	18	18	2	-14	-2	-2	-2	-6	
X.24	3542	-1386	406	-42	70	6	5	-42	6	22	-10	2	2	-3	15	-3	-9	1	3	1	3	1	3	
X.25	3542	1078	182	-42	70	-26	14	42	-10	14	-6	2	-6	-2	2	6	-2	2	-6	-2	-2	-2	
X.26	3542	-1386	406	-42	70	6	5	-42	6	22	-10	2	2	-3	15	-3	9	1	3	1	-3	1	3	
X.27	3795	-1485	435	-45	83	-13	15	-41	-13	11	7	3	-5	-1	15	-9	-9	3	3	-1	1	1	
X.28	3795	-1485	435	-45	-13	19	15	-1	19	11	-17	-5	3	-1	15	-9	9	3	3	-1	1	1	
X.29	5313	-2079	609	-63	49	17	-15	-35	17	25	-19	-3	5	1	3	15	9	-9	-3	3	1	-1	
X.30	7084	2156	364	-84	140	76	19	28	-20	-4	-4	4	4	-1	21	3	5	-5	-3	-1	1	1	
X.31	8855	-3465	1015	-105	7	39	-10	-21	39	31	-37	3	-5	-1	6	-2	
X.32	10120	3080	520	-120	168	40	-5	56	-24	8	-8	-15	3	-7	7	-3	3	1	-2	-2	
X.33	10626	3234	546	-126	-14	-46	6	14	2	10	-2	6	2	1	-6	6	-6	6	-6	-2	2	
X.34	10626	3234	546	-126	-14	-46	-3	14	2	10	-2	6	2	1	3	-3	3	-3	3	1	-1	
X.35	10626	3234	546	-126	-14	-46	-3	14	2	10	-2	6	2	1	3	-3	3	-3	3	1	-1	
X.36	11385	-4455	1305	-135	-87	9	45	9	-15	-3	5	-3	1	
X.37	11385	-4455	1305	-135	-87	9	45	9	-15	-3	5	-3	1	
X.38	12880	-560	-560	80	112	-16	10	16	-16	8	8	-8	-10	2	-2	2	2	-2	2	
X.39	12880	-560	-560	80	-112	16	10	-16	16	-10	2	-2	2	2	2	-2	2	
X.40	12880	-560	-560	80	-112	16	10	-16	16	-10	2	-2	2	2	2	-2	2	
X.41	14168	-616	-616	88	168	-24	20	24	-24	-4	-4	4	3	-20	4	-4	-4	4	
X.42	14168	4312	728	-168	56	-72	11	56	-8	24	-8	-2	9	3	1	-1	-3	-1	-3	
X.43	17710	5390	910	-210	-98	62	25	-70	14	-26	10	-6	2	-2	15	9	-1	-9	1	-1	
X.44	20608	-896	-896	128	-20	3	20	-4	4	4	-4		
X.45	20608	-896	-896	128	-20	3	20	-4	4	4	-4		
X.46	22770	-8910	2610	-270	162	-30	-78	-30	18	18	-2	-2	-2	
X.47	22770	6930	1170	-270	-126	-30	-42	18	-6	6	6	-2	2	
X.48	22770	6930	1170	-270	-126	-30	-42	18	-6	6	6	-2	2	
X.49	26565	-10395	3045	-315	-91	5	15	49	5	-19	1	-3	5	1	15	-9	-9	3	3	-1	-1	
X.50	28336	8624	1456	-336	112	112	-14	-16	-16	1	-6	6	2	-2	6	-2	2	
X.51	30360	-11880	3480	-360	-8	-8	-15	8	-8	-8	8	-15	9	9	-3	-3	1	1	1	1	
X.52	32384	9856	1664	-384	-16	-1	-24	
X.53	35420	10780	1820	-420	28	-36	5	28	-4	12	-4	-4	-4	-4	15	-3	7	-7	3	1	3	

Character table of $E(\text{Fi}_{23})$ (continued)

2	.	.	2	2	1	1
3	1	1
5	1	1
7	.	.	1	1	.	.
11
23	1	1
	23a	23b	28a	28b	30a	30b
2P	23a	23b	14a	14b	15a	15b
3P	23a	23b	28b	28a	10a	10a
5P	23b	23a	28b	28a	6a	6a
7P	23b	23a	4a	4a	30b	30a
11P	23b	23a	28a	28b	30b	30a
23P	1a	1a	28a	28b	30a	30b
X.1	1	1	1	1	1	1
X.2	-1	-1	-1	-1	-1	-1
X.3	-1	-1	-A	-A	.	.
X.4	-1	-1	-A	-A	.	.
X.5	.	.	1	1	.	.
X.6	1	1	.	.	1	1
X.7	1	1	.	.	B	B
X.8	1	1	.	.	B	B
X.9	.	.	-1	-1	1	1
X.10	.	.	-1	-1	.	.
X.11	-1	-1
X.12	C	C
X.13	C	C
X.14	-1	-1	.	.	1	1
X.15	-1	-1	.	.	1	1
X.16	1	1	A	A	.	.
X.17	1	1	A	A	.	.
X.18	.	.	-1	-1	.	.
X.19
X.20	.	.	1	1	.	.
X.21	.	.	1	1	-1	-1
X.22	.	.	-A	-A	.	.
X.23	.	.	-A	-A	.	.
X.24
X.25	1	1
X.26
X.27	.	.	1	1	.	.
X.28	.	.	-1	-1	.	.
X.29
X.30	1	1
X.31
X.32
X.33	-1	-1
X.34	-B	-B
X.35	-B	-B
X.36	.	.	A	A	.	.
X.37	.	.	A	A	.	.
X.38
X.39
X.40
X.41
X.42	-1	-1
X.43
X.44
X.45
X.46	.	.	-1	-1	.	.
X.47
X.48
X.49
X.50	-1	-1
X.51	.	.	1	1	.	.
X.52	1	1
X.53
X.54
X.55
X.56

$$A = \zeta(7)_7^4 + \zeta(7)_7^2 + \zeta(7)_7, B = -2\zeta(15)_3\zeta(15)_5^3 - 2\zeta(15)_3\zeta(15)_5^2 - \zeta(15)_3 - \zeta(15)_5^3 - \zeta(15)_5^2 - 1, C = \zeta(23)_{23}^{18} + \zeta(23)_{23}^{16} + \zeta(23)_{23}^{13} + \zeta(23)_{23}^{12} + \zeta(23)_{23}^9 + \zeta(23)_{23}^8 + \zeta(23)_{23}^6 + \zeta(23)_{23}^4 + \zeta(23)_{23}^3 + \zeta(23)_{23}^2 + \zeta(23)_{23}, D = \zeta(11)_{11}^9 + \zeta(11)_{11}^5 + \zeta(11)_{11}^4 + \zeta(11)_{11}^3 + \zeta(11)_{11}, E = -2\zeta(8)_8^3 - 2\zeta(8)_8, F = 2\zeta(7)_7^4 + 2\zeta(7)_7^2 + 2\zeta(7)_7.$$

Character table of $D(\text{Fi}_{23})$ (continued)

	2	3	1	1	5	5	4	4	2	2	2	2	2	2	5	5	1	1
3	1	1	1	1
5	1	1	1	1	1	1
7	1	1	1	1	1	1
11	1	1	1	1	1	1	.	.	1	1
10g	11a	11b	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	16a	16b	22a	22b		
2P	5a	11b	11a	6a	6a	6b	6b	7a	7b	7a	7b	7a	7b	8a	8a	11a	11b	
3P	10g	11a	11b	4f	4e	4a	4b	14f	14e	14d	14c	14b	14a	16a	16b	22a	22b	
5P	2a	11a	11b	12a	12b	12c	12d	14f	14e	14d	14c	14b	14a	16b	16a	22a	22b	
7P	10g	11b	11a	12a	12b	12c	12d	2b	2a	2c	2c	2a	2b	16b	16a	22b	22a	
11P	10g	1a	1a	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	16a	16b	2a	2a	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.3	.	1	1	A	A	A	A	A	A	-1	-1	1	1	
X.4	.	1	1	A	A	A	A	A	A	-1	-1	1	1	
X.5	.	.	.	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	.	.	
X.6	2	.	.	1	1	-1	-1	-1	-1	.	.	
X.7	-1	1	1	1	1	1	1	1	-1	-1	.	
X.8	-1	.	.	1	1	1	1	
X.9	-1	.	.	-1	1	-1	1	-1	-1	1	1	-1	-1	
X.10	-1	.	.	-1	1	1	-1	1	-1	-1	-1	-1	1	
X.11	.	1	1	-1	-1	-1	-1	1	1	
X.12	1	.	.	1	1	1	1	-1	-1	.	.	
X.13	.	B	B	1	1	1	1	B	B	
X.14	.	B	B	1	1	1	1	B	B	
X.15	.	.	.	-2	-2	.	.	-1	1	-1	-1	1	-1	
X.16	.	.	.	2	2	-2	-2	1	1	.	.	
X.17	.	.	.	-2	-2	-2	-2	1	1	.	.	
X.18	.	.	.	-1	-1	1	1	1	1	.	.	
X.19	1	
X.20	1	
X.21	1	.	.	-1	-1	1	1	
X.22	1	.	.	-1	-1	1	1	
X.23	-2	1	1	2	-2	-1	-1	
X.24	-2	-1	-1	
X.25	.	.	.	1	1	-1	-1	
X.26	-A	A	-A	-A	A	-A	
X.27	-A	A	-A	-A	A	-A	
X.28	-1	.	.	1	-1	1	-1	1	1	-1	-1	1	1	
X.29	-1	.	.	1	-1	1	-1	1	1	1	1	1	-1	
X.30	.	.	.	-1	-1	1	1	1	1	.	.	
X.31	.	.	.	-1	-1	1	1	-1	-1	.	.	
X.32	.	.	.	-1	1	1	-1	A	-A	-A	-A	-A	A	
X.33	.	.	.	-1	1	-1	1	-A	-A	A	A	-A	-A	
X.34	.	.	.	-1	1	1	-1	A	-A	-A	-A	-A	A	
X.35	.	.	.	-1	1	-1	1	-A	-A	A	A	-A	-A	
X.36	1	-1	1	1	-1	1	
X.37	.	.	.	2	2	
X.38	.	.	.	1	1	-1	-1	
X.39	.	.	.	2	2	
X.40	.	.	.	-2	-2	
X.41	1	.	.	1	-1	1	-1	
X.42	1	.	.	-2	2	2	-2	
X.43	1	.	.	-2	2	-2	2	
X.44	1	.	.	1	-1	-1	1	
X.45	.	.	.	1	-1	-1	1	1	-1	-1	-1	-1	1	
X.46	.	.	.	2	2	.	.	-1	1	-1	-1	1	-1	
X.47	.	.	.	1	-1	1	-1	-1	1	1	-1	-1	
X.48	.	B	B	2	-2	-B	-B	
X.49	.	B	B	2	-2	-B	-B	
X.50	-1	-1	.	.	
X.51	1	1	.	.	
X.52	-1	.	.	-1	1	-1	1	
X.53	-1	.	.	-1	1	1	-1	
X.54	.	.	.	-2	-2	
X.55	-1	
X.56	-1	
X.57	.	.	.	1	-1	1	-1	
X.58	
X.59	.	.	.	1	-1	-1	1	
X.60	D	D	
X.61	D	D	
X.62	.	-1	-1	-2	2	1	1	
X.63	.	-1	-1	2	-2	1	1	
X.64	
X.65	
X.66	
X.67	-2	.	.	-2	2	
X.68	1	1	1	-1	-1	
X.69	1	-1	-1	

$$A = -\zeta(7)_7^4 - \zeta(7)_7^2 - \zeta(7)_7 - 1, B = -\zeta(11)_{11}^9 - \zeta(11)_{11}^5 - \zeta(11)_{11}^4 - \zeta(11)_{11}^3 - \zeta(11)_{11} - 1, C = 2\zeta(5)_5^3 + 2\zeta(5)_5^2 + 1, D = -2\zeta(8)_8^3 - 2\zeta(8)_8.$$

Character table of $H(F_{i_{23}})$ (continued)

	2	4	9	8	8	8	7	8	8	4	8	8	9	9	5	5	8	8	7	7	7	6	6
	3	1	7	9	5	5	7	6	6	7	4	4	3	3	5	5	3	3	3	3	3	3	3
	5	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2P	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
	3P	5a	3a	3b	3a	3c	3b	3b	3d	3b	3b	3a	3a	3c	3c	3c	3b	3b	3c	3c	3a	3c	3c
	5P	5a	2a	2a	2b	2c	2a	2c	2b	2a	2d	2e	2d	2e	2b	2c	2f	2f	2f	2f	2f	2e	2d
	7P	1a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
	7P	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
	11P	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
	13P	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	3	15	-3	-7	-7	6	-7	-7	-3	5	-1	-1	2	2	1	1	1	-2	-2	-2	1	2	2
X.3	2	8	-28	.	.	-10	.	.	-1	4	-4	-8	8	8	.	.	.	6	-6	7	7	2	-2
X.4	4	6	24	14	14	15	-4	-4	-3	12	6	6	5	5	4	4	4	7	7	7	-2	3	3
X.5	1	56	29	-6	-6	2	-15	-15	2	5	9	8	8	-6	-6	-7	-7	2	2	2	2	2	2
X.6	5	-1	-28	-19	-19	26	8	8	-1	-4	-4	-1	-1	8	8	.	.	-6	-6	-3	3	2	2
X.7	5	-64	26	-24	-24	-19	6	-6	-1	-6	6	.	.	3	-3	6	-6	-3	3	3	3	-3	-3
X.8	5	-64	26	-24	-24	-19	-6	6	-1	-6	6	.	.	-3	3	-6	6	-3	3	3	3	3	-3
X.9	3	105	6	17	17	15	26	26	6	14	14	-7	-7	-1	-1	2	2	-1	-1	-7	-1	-1	-1
X.10	5	119	2	31	31	20	22	22	2	10	10	7	7	4	4	-2	-2	4	4	7	4	4	4
X.11	-1	-154	8	-4	4	8	-32	32	8	8	-8	2	-2	-4	-4	4	-4
X.12	-1	-154	8	4	-4	8	32	-32	8	8	-8	2	-2	4	-4	4	-4
X.13	-1	-154	8	4	-4	8	32	-32	8	8	-8	2	-2	4	-4	4	-4
X.14	-1	-154	8	-4	4	8	-32	32	8	8	-8	2	-2	-4	-4	4	-4
X.15	.	15	114	5	5	24	14	14	6	-6	-6	15	15	-4	-4	-10	-10	8	8	5	.	.	.
X.16	.	105	123	25	25	33	7	7	15	3	3	9	9	7	7	15	15	9	9	9	9	-3	-3
X.17	3	-120	-120	-40	-40	-39	4	-4	-12	-8	8	-8	8	-13	13	-12	12	9	9	-9	-9	-1	-1
X.18	3	-120	-120	-40	-40	-39	-4	4	-12	-8	8	-8	8	13	-13	12	-12	9	9	-9	-9	-1	-1
X.19	6	120	84	.	.	-60	-34	-34	3	20	-20	8	-8	-12	12	.	.	4	-4
X.20	5	-21	-102	29	29	60	-34	-34	6	-14	-14	-5	-5	2	2	-10	-10	-4	-4	5	4	4	4
X.21	7	91	-44	-79	-79	64	20	20	10	4	4	-5	-5	2	2	4	4	-8	-8	1	4	4	4
X.22	5	399	-60	71	71	48	-44	-44	-6	20	20	-1	-1	10	10	4	4	-8	-8	1	4	4	4
X.23	-5	441	90	-21	-21	-18	42	42	9	-6	-6	9	9	6	6	10	10	-2	-2	19	6	6	6
X.24	-2	-546	-96	-4	4	3	-68	68	-15	16	-16	10	-10	5	-5	12	-12	3	-3	5	5	-5	-5
X.25	-2	-546	-96	-4	4	3	68	-68	-15	16	-16	10	-10	-5	5	-12	12	3	-3	5	5	-5	-5
X.26	-2	-84	258	-44	-44	6	10	10	-12	18	18	12	12	-8	-8	-6	-6	6	6	-12	6	6	6
X.27	.	-35	235	5	5	19	-13	-13	-8	-5	-5	13	13	5	5	3	3	3	3	3	3	-5	-5
X.28	.	595	73	-35	-35	10	-71	-71	19	9	9	3	3	-8	-8	-7	-7	2	2	5	6	6	6
X.29	.	-210	150	30	30	51	-6	-6	-12	-2	-2	-2	-2	3	3	2	2	11	11	-10	7	7	7
X.30	.	735	-93	25	25	-3	79	79	-12	11	11	-1	-1	-11	-11	-9	-9	-3	-3	9	5	5	5
X.31	.	420	-12	-10	-10	42	-64	-64	15	28	28	4	4	-10	-10	8	8	2	2	-10	-2	-2	-2
X.32	6	.	162	90	90	81	-18	-18	.	-6	-6	.	.	9	9	6	6	9	9	-6	-3	-3	-3
X.33	.	-120	24	.	.	-48	.	.	24	-8	8	-8	8	8	-8	-8
X.34	.	-120	24	.	.	-48	.	.	24	-8	8	-8	8	8	-8	-8
X.35	.	685	28	-25	-25	-8	-52	-52	-26	12	12	-3	-3	2	2	-4	-4
X.36	.	280	-404	.	.	-62	.	.	28	12	-12	-24	24	18	-18	.	.	6	-6
X.37	6	.	108	.	.	54	.	.	-54	12	-12	6	-6	.	.	-6	-6
X.38	.	630	153	30	30	-45	-33	-33	9	-9	-9	6	6	3	3	15	15	3	3	6	3	3	3
X.39	.	630	-153	30	30	-45	-33	-33	9	-9	-9	6	6	3	3	15	15	3	3	6	3	3	3
X.40	7	-776	16	104	-104	-56	-32	32	16	-16	16	8	-8	-4	-4	4	-4	-4
X.41	7	-776	16	-104	104	-56	-32	32	16	-16	16	8	-8	4	4	4	-4	-4
X.42	.	525	57	55	55	48	73	73	-24	25	25	13	13	-8	-8	9	9	.	.	-9	-8	-8	-8
X.43	-5	-279	-144	-69	-69	72	-24	-24	18	.	.	9	9	12	12	-8	-8	-8	-8	-5	.	.	.
X.44	.	160	-380	.	.	52	.	.	-29	4	-4	-32	32	-12	12	.	.	-4	4
X.45	-3	.	324	-72	-72	-81	-36	36	.	-12	12	.	.	9	-9	12	-12	-9	9	-3	3	3	3
X.46	-3	.	324	-72	-72	-81	-36	-36	.	-12	12	.	.	-9	9	-12	12	-9	9	-3	3	3	3
X.47	.	405	162	-45	-45	-81	-18	-18	.	-6	-6	21	21	9	9	6	6	-9	-9	3	3	3	3
X.48	.	-210	-291	-50	-50	114	49	49	-21	-11	-11	-2	-2	4	4	9	9	-6	-6	6	-2	-2	-2
X.49	-5	406	-116	114	114	64	-12	-12	-8	12	12	6	6	6	-12	-12	.	.	-6
X.50	-5	-406	116	-44	-44	-64	-28	28	8	36	-36	-18	18	10	-10	-12	12
X.51	-5	-406	116	44	-44	-64	28	-28	8	36	-36	-18	18	-10	10	12	-12
X.52	5	729	.	81	81	9	9	9	.	.	.
X.53	6	405	324	-45	-45	.	36	36	.	-12	-12	21	21	-18	-18	-12	-12	.	.	3	.	.	.
X.54	.	-1610	64	20	-20	37	124	-124	-17	16	-16	-14	14	-7	7	12	-12	-3	3	-8	-1	-1	-1
X.55	.	-280	-307	40	40	-10	-23	-23	17	5	5	8	8	-14	-14	1	1	-2	-2	.	.	2	2
X.56	.	-1610	64	-20	20	37	-124	124	-17	16	-16	-14	14	7	-7	-12	12	-3	3	.	.	-1	-1
X.57	.	-280	-307	40	40	-10	-23	-23	17	5	5	8	8	-14	-14	1	1	-2	-2	-8	2	2	2
X.58	.	980	98	60	60	-28	-66	-66	-10	-6	-6	-12	-12	6	6	-10	-10	-4	-4
X.59	.	980	98	60	60	-28	-66	-66	-10	-6	-6	-12	-12	6	6	-10	-10	-4	-4
X.60	.	-840	-210	-80	80	42	-46	46	33	14	-14	8	-8	8	8	18	-18	-6	6	.	.	-2	2
X.61	.	-840	-210	80	-80	42	46	-46	33	14	-14	8	-8	8	-8	-18	18	-6	6	.	.	-2	2
X.62	.	1575	171	-15	-15	-18	111	111	9	3	3	-9	-9	12	12	7	7	-2	-2	-23	6	6	6
X.63	.	-315	-72	45	45	9	-36	-36	9	.	.	21	21	-9	-9	4	4	1	1	-11	-3	-3	-3
X.64	5	336	174	-136	136	-69	-																

Character table of $H(\text{Fi}_{23})$ (continued)

	2	5	5	4	4	2	7	7	7	6	6	3	2	1	4	3	3	4	4	2	2	7	7	5	6	6	6	5	5	
	3	3	3	3	3	1	1	1	1	6	6	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	2P	3c	3c	3d	3d	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	
	3P	2f	2f	2f	2f	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	
	5P	622	623	624	625	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	
	7P	622	623	624	625	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	
	11P	622	623	624	625	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	
	13P	622	623	624	625	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11b	11a	12a	12b	12c	12d	12e	12f	12g	12h	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	-2	-2	1	1	1	1	2	-2	2	.	3	.	.	.	3	1	1	-1	-1	1	1	3	3	-3	-3	-3	-3	-1	-1	
X.3	.	.	-3	3	2	2	1	1	1	1	-2	4	1	-2	.	.	.	2	-2	2
X.4	1	1	1	1	2	1	1	1	1	1	2	3	.	.	4	2	2	-2	-2	4	4	1	-2	-2	2	
X.5	2	2	2	2	.	-3	1	1	-1	-1	2	3	.	.	-1	-1	1	1	1	.	.	.	4	4	1	-2	-2	-1	-1	
X.6	.	.	3	3	2	-2	2	2	.	.	-1	-2	5	1	1	1	1	1	1	.	.	.	-1	-1	-4	-3	-3	5	-2	
X.7	-3	3	-3	3	1	4	-2	1	-5	1	-1	1	1	1	-4	-4	
X.8	3	-3	-3	3	1	4	-2	1	-5	1	1	1	1	1	4	-4	
X.9	-1	-1	2	2	.	-1	-1	3	-1	-1	3	5	5	2	5	5	5	.	
X.10	4	4	-2	-2	.	4	4	.	.	.	5	-1	-1	5	1	1	1	1	1	.	.	.	3	3	6	3	3	.	.	
X.11	1	1	1	1	1	-1	-1	1	A	A	6	-6	.	-2	-2	.	.	.	
X.12	1	1	1	1	-1	-1	-1	1	A	A	6	-6	.	-2	-2	.	.	.	
X.13	1	1	1	1	-1	-1	-1	1	A	A	6	-6	.	-2	-2	.	.	.	
X.14	1	1	1	1	-1	-1	-1	1	A	A	6	-6	.	-2	-2	.	.	.	
X.15	-4	.	.	.	1	1	-3	1	-1	-1	.	3	-1	-1	1	1	1	1	1	1	1	1	
X.16	3	3	3	3	.	-2	-2	-2	2	2	.	3	-1	-1	1	1	1	1	1	1	1	1	
X.17	-3	-3	.	.	3	3	3	.	.	-3	-3	3	-1	1	-2	2	
X.18	3	-3	.	.	1	3	.	.	-3	-3	3	-1	1	-2	2	
X.19	.	.	-3	3	2	-6	.	.	.	-6	-2	2
X.20	2	2	2	2	.	2	-2	2	.	.	-3	-3	.	.	5	-1	-1	1	1	.	.	-1	-1	2	5	5	-7	.	.	
X.21	-2	-2	-2	-2	.	4	-4	.	.	.	1	1	-2	7	1	1	-1	-1	-5	-5	-8	-3	-3	1	.	
X.22	-2	-2	-2	-2	.	-4	4	.	.	.	3	3	.	.	5	-1	-1	1	1	-1	-1	3	3	.	-3	-3	-3	.	.	
X.23	-2	-2	1	1	.	1	5	1	-1	-1	.	3	.	.	-5	-1	-1	-1	-1	.	.	.	5	5	2	-1	-1	-1	-2	
X.24	3	-3	3	-3	2	-2	2	2	2	-2	-2	-2	6	-6	.	-2	-2	2	-2	
X.25	-3	3	3	-3	2	-2	2	2	2	-2	-2	-2	6	-6	.	-2	-2	2	-2	
X.26	3	.	.	.	-2	-2	-2	2	2	.	.	.	-4	-4	2	.	-4	-2	3	
X.27	-3	-3	.	.	2	2	-2	-2	-2	1	1	1	1	1	-5	1	1	1	3	3
X.28	-4	-4	-1	-1	.	2	-2	-2	.	.	4	-2	1	7	7	1	1	1	1	1	1	
X.29	-1	-1	-4	-4	-6	2	2	-2	-2	-2	2	2	
X.30	-3	-3	.	.	-5	-1	-1	1	1	1	3	7	7	-5	1	1	1	1	2	2
X.31	-2	-2	-1	-1	.	-1	3	3	1	1	-3	-3	4	4	4	-6	-6	-2	2	2	
X.32	-3	-3	.	.	-7	-3	1	-1	-1	6	.	.	-2	-2	6	2	2	-6	.	.	
X.33	-2	-3	
X.34	-2	-3	
X.35	2	2	2	2	-1	4	-4	.	.	.	4	-2	1	1	1	-8	-1	-1	-5	.	.	
X.36	-4	-2	-1	-2	-2
X.37	.	.	6	-6	-6	.	.	-2	2	-2	-2
X.38	3	3	-3	-3	2	2	-1	2	2	2	-1	-1	
X.39	3	3	-3	-3	2	2	-1	2	2	2	-1	-1	
X.40	-1	5	-1	-1	-7	-1	1	-1	1	1	
X.41	-1	5	-1	-1	-7	1	-1	-1	1	1	
X.42	2	-2	2	.	.	.	-3	5	5	-7	3	3	-1	-3	-3	
X.43	4	4	-2	-2	1	-4	4	-5	1	1	-1	-1	.	.	1	1	-8	3	3	-5	2	-2	
X.44	.	.	-3	3	-2	2	2	-1	-2	-2
X.45	3	-3	3	1	-1	1	-1	-1	-1	-1	
X.46	-3	3	.	.	-1	-1	-5	3	1	1	.	3	-1	1	1	-1	-1	-1	-1	
X.47	-3	-3	.	.	-1	-1	-5	3	1	1	.	3	-1	1	1	-1	-1	-1	-1	
X.48	.	.	3	3	-4	-3	1	-5	-1	-1	-1	-1	.	.	.	-3	-3	6	-1	-1	3	-1	-1	
X.49	6	6	4	-2	1	-5	-1	-1	-1	-1	.	.	.	-6	-6	.	-2	-2	6	.	
X.50	6	-6	4	-2	1	-5	-1	-1	-1	-1	.	.	.	-6	-6	.	-2	-2	6	.	
X.51	-6	6	4	-2	1	-5	-1	-1	-1	-1	.	.	.	-6	-6	.	-2	-2	6	.	
X.52	-2	-1	-1	-1	-1	-1	.	5	1	1	1	1	1	1	1	.	.	.	-3	-3	.	-3	-3	-3	.	
X.53	6	6	.	.	-1	-4	4	6	-2	-2	.	.	.	-3	-3	.	-1	-1	3	.	
X.54	3	-3	-3	3	2	-2	-1	-1	-1	6	-6	.	2	-2	.	-2	2	
X.55	-2	-2	1	1	-1	-1	-1	-3	-5	-5	
X.56	-3	3	-3	3	-2	2	-1	-1	-1	6	-6	.	-2	2	.	-2	2	
X.57	-2	-2	1	1	-1	-1	-1	-3	7	7	
X.58	2	2	2	2	-4	2	-1	4	4	-2	.	.	4	.	
X.59	2	2	2	2	-4	2	-1	4	4	-2	.	.	4	.	
X.60	.	.	3	-3	-3	1	1	-2	-2
X.61	4	4	.	.	3	-3	-3	1	1	-4	4	
X.62																														

Character table of $H(F_{i_{23}})$ (continued)

	2	5	5	5	5	4	4	5	5	3	3	1	1	2	2	2	2	5	5	3	2	3	3	3	3	2	2	3
	3	2	2	2	2	2	2	1	1	2	2	.	.	1	.	.	.	1	.	4	4	3	3	3	3	3	3	2
5
7
11
13
	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$12p$	$12q$	$12r$	$13a$	$13b$	$14a$	$14b$	$14c$	$15a$	$16a$	$16b$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18g$	$18h$	$18i$	
2P	6_{16}	6_{18}	6_{18}	6_{16}	6_{20}	6_{20}	6_{12}	6_9	6_{24}	6_{24}	$13b$	$13a$	$7a$	$7a$	$7a$	$15a$	$8c$	$8c$	$9a$	$9b$	$9a$	$9a$	$9a$	$9a$	$9b$	$9b$	$9a$	
3P	$4f$	$4e$	$4f$	$4e$	$4b$	$4c$	$4g$	$4d$	$4f$	$4e$	$13a$	$13b$	$14a$	$14b$	$14c$	$5a$	$16a$	$16b$	6_2	6_2	6_6	6_7	6_6	6_7	6_6	6_7	6_{10}	
5P	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$12p$	$12q$	$12r$	$13b$	$13a$	$14a$	$14b$	$14c$	$3a$	$16b$	$16a$	$18a$	$18b$	$18c$	$18f$	$18c$	$18d$	$18g$	$18h$	$18i$	
7P	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$12p$	$12q$	$12r$	$13b$	$13a$	$2a$	$2b$	$2c$	$15a$	$16b$	$16a$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18c$	$18d$	$18h$	$18i$
11P	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$12p$	$12q$	$12r$	$13b$	$13a$	$14a$	$14b$	$14c$	$15a$	$16a$	$16b$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18c$	$18d$	$18h$	$18i$
13P	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$12p$	$12q$	$12r$	$1a$	$1a$	$14a$	$14b$	$14c$	$15a$	$16b$	$16a$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18c$	$18d$	$18h$	$18i$
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	-1	2	2	-1	.	.	-1	1	-1	-1	.	.	1	1	1	.	.	.	3	.	-1	-1	-1	-1	2	2	-1	
X.3	2	-2	2	-2	-1	1	1	-2	.	.	.	2	.	.	2	-4	2
X.4	2	-1	-1	-2	-1	-1	2	.	-1	-1	.	.	2	.	.	1	-1	-1	3	2	2	2	2	-1	-1	.	.	
X.5	-1	2	2	-1	-2	-2	.	-3	2	2	1	1	1	2	2	
X.6	-2	-2	-2	-2	1	-1	.	-1	1	1	.	.	2	.	.	-1	.	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.7	-4	1	-1	4	1	-1	.	-1	1	1	.	.	2	.	.	1	-1	-1	-4	2	
X.8	2	1	-1	-2	-1	1	.	-1	1	1	.	.	-1	-1	1	-1	.	.	-4	2	
X.9	.	3	3	.	-1	-1	1	-2	-1	-1	3	.	-1	-1	-1	-1	2	2	.	.	
X.10	2	-2	-1	.	.	5	-1	1	1	1	1	1	1	1	
X.11	-2	-1	-1	.	.	.	-1	.	.	-1	-1	1	-1	-1	-1	-1	-1	-1	
X.12	-2	2	-1	.	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.13	-2	2	-1	.	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.14	2	-2	-1	.	.	-1	-1	1	-1	-1	-1	-1	-1	-1	
X.15	-1	-2	1	-1	-1	.	-1	-1	.	3	2	2	2	-1	-1	.	.	
X.16	1	1	1	1	1	1	1	-1	1	1	3	-2	-2	-2	-2	1	1	1	1	
X.17	2	1	-1	-2	1	-1	.	.	2	-2	.	.	-1	-1	1	.	.	.	-3	1	-1	-1	-1	-1	-1	-1	-1	
X.18	2	1	-1	-2	-1	1	.	.	2	-2	.	.	-1	1	-1	.	.	.	-3	-3	-1	1	-1	1	-1	1	-1	
X.19	-2	-4	4	2	1	-1	.	.	-2	6	-2	
X.20	2	2	-1	-2	-1	.	.	-3	-3	-1	-1	-1	-1	-1	-1	-1	
X.21	-1	-1	.	.	1	1	-1	-1	-1	-1	-1	-1	-1	
X.22	-1	-1	.	.	3	3	1	1	1	1	1	1	-1	
X.23	-2	-2	-2	-2	2	2	1	2	-1	-1	-1	
X.24	-2	-1	1	2	-1	1	.	.	1	-1	-3	1	-1	-1	-1	-2	2	-1		
X.25	-2	-1	1	2	-1	-1	.	.	1	-1	-3	-1	1	-1	-1	-2	-2	-1		
X.26	-2	-2	-2	-2	2	2	.	.	2	-2	-2	1	1	-1	-1	-1	-1	-1	-1	-1	
X.27	3	3	3	3	1	1	-3	3	1	1	-1	-1	-1	-1	-1	-1	1	
X.28	1	-2	-2	1	-2	-2	3	1	1	1	4	-2	-2	-2	-2	-2	-2	-2	-2	
X.29	.	3	3	.	1	1	2	-2	1	1	-6	-2	
X.30	1	1	1	1	1	1	-1	-1	-2	-2	1	1	3	.	1	1	1	1	-2	
X.31	2	2	2	2	.	.	.	-1	-1	-1	-1	-3	-3	-1	-1	-1	-1	-1	
X.32	.	-3	-3	.	-1	-1	.	2	1	1	1	
X.33	1	1	2	3	.	\bar{B}	\bar{B}	\bar{B}	\bar{B}	.	.	-1	
X.34	1	1	2	3	.	\bar{B}	\bar{B}	\bar{B}	\bar{B}	.	.	-1	
X.35	2	2	1	.	-2	-2	-1	-1	.	-1	-1	-1	.	.	4	-2	2	2	2	2	2	2	.	
X.36	-2	2	-2	2	.	.	.	-2	2	-1	-1	4	-2	
X.37	-2	2	-2	2	.	.	.	-2	2	1	1	4	-2	
X.38	-1	-1	-1	-1	-1	-1	-2	-1	-1	-1	
X.39	-1	-1	-1	-1	-1	-1	-2	-1	-1	-1	C	D	
X.40	-2	-2	1	1	-1	1	-5	1	1	-1	1	-1	1	-1	1	
X.41	-2	-2	1	1	-1	1	-5	1	-1	1	-1	1	-1	1	1	
X.42	-3	.	-3	.	1	1	1	-1	-1	1	-3	.	1	1	1	-2	-2	1	1	
X.43	1	1	2	-1	-1	1	-2	-2	2	
X.44	-2	-4	4	2	1	-1	.	.	2	
X.45	.	-3	-3	.	1	-1	.	.	1	-1	
X.46	.	-3	-3	.	-1	1	
X.47	.	-3	-3	.	-1	-1	1	2	.	.	-1	-1	-1	1	1	
X.48	-1	2	2	-1	-2	-2	2	1	-1	-1	.	.	-1	-1	-1	.	.	.	-3	-3	1	1	1	1	1	1	1	
X.49	-2	-2	4	-2	
X.50	-2	-2	-4	2	2	-2	2	-2	-2	-2	
X.51	2	2	-4	2	-2	2	-2	2	-2	-2	
X.52	2	2	1	.	.	.	1	1	-2	.	-1	-1	-1	
X.53	2	2	-1	1	1	
X.54	2	1	-1	-2	1	-1	.	.	-1	1	1	1	1	1	-1	-1	1	-1	-1	-1	
X.55	7	-2	-2	7	.	.	.	1	1	1	1	-1	-1	1	1	1	1	1	1	-1	
X.56	2	1	-1	-2	-1	1	.	.	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	
X.57	-5	-2	-2	-5	.	.	.	1	1	1	-1	-1	1	1	1	1	1	1	-1	
X.58	2	2
X.59	2	2	\bar{E}	\bar{E}	-4	2	
X.60	4	2																										

Character table of $H(F_{i_{23}})$ (continued)

	2	3	5	7	11	13	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
2P	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	2d	2e	2f	1a	1a	1a	1a	4a	4b	4c	4d	4e	4f	4g
3P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g						
5P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g						
7P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g						
11P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g						
13P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g						
X.88	1201200	1201200	-30800	-30800	560	560	176	420	-30	-48	51	-16												
X.89	1201200	-1201200	-24640	24640	-1680	1680		-210	132	-39	24													
X.90	1297296	-1297296	44352	-44352	-1200	1200		1134	-324	-81														
X.91	1297296	-1297296	-44352	44352	-1200	1200		1134	-324	-81														
X.92	1360800	1360800	30240	30240	1440	1440	288	1560	486															
X.93	1372800	1372800	49280	49280	640	640	128	-512	640	64	-8													
X.94	1441792	1441792						-512	640	64	-8													
X.95	1441792	-1441792						-512	640	64	-8													
X.96	1663200	-1663200			2400	-2400			108	54	27													
X.97	1663200	1663200			2400	-2400			108	54	27													
X.98	1791153	1791153	-5103	-5103	-2511	-2511	81																	
X.99	1876446	1876446	37422	37422	-1890	-1890	-306	729																
X.100	2027025	2027025	-24255	-24255	-1455	-1455	33																	
X.101	2050048	2050048			2048	2048		-1232	-224	-80	-8													
X.102	2196480	-2196480	11264	-11264	1024	-1024		-456	-240	-24	-24													
X.103	2196480	-2196480	-11264	11264	1024	-1024		-456	-240	-24	-24													
X.104	2316600	2316600	-43560	-43560	-840	-840	24	405	-162															
X.105	2358720	-2358720	8064	-8064	-1344	1344		-1134	-324															
X.106	2358720	-2358720	-8064	8064	-1344	1344		-1134	-324															
X.107	2402400	2402400	12320	12320	-160	-160	160	-735	-384	48	-6	-64	-40	-40										
X.108	2402400	-2402400	49280	-49280	1760	-1760		840	-60	66	21													
X.109	2402400	-2402400	-49280	49280	-1760	1760		840	-60	66	21													
X.110	2555904	2555904	-32768	-32768				-384	192	-96	-24													
X.111	2555904	-2555904	32768	32768				-384	192	-96	-24													
X.112	2555904	-2555904	-32768	32768				-384	192	-96	-24													
X.113	2555904	2555904	32768	-32768				-384	192	-96	-24													
X.114	2729376	2729376	7776	7776	-864	-864	-288	-729																

Character table of $H(F_{i_{23}})$ (continued)

	2	3	5	7	11	13	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂		
2P	5a	3a	3b	3a	3c	3b	3b	3d	3b	3b	3a	3a	3c	3c	3b	3b	3c	3c	3a	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	
3P	5a	2a	2a	2b	2c	2c	2a	2c	2b	2a	2d	2e	2e	2b	2c	2f	2f	2f	2f	2f	2f	2f	2e	2d	2f						
5P	1a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂								
7P	1a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂								
11P	1a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂								
13P	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂								
X.88		420	-30	-20	-48	34	34	51	2	2	-28	-28	-2	-2	2	2	8	8	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	
X.89		210	-132	20	-20	39	-92	92	-24	12	-12	6	-6	11	-11	-12	12	-9	9												
X.90		-4	-1134	324	-36	36	81	-36	-36		-12	12	6	-6	9	-9	12	12	9	-9											
X.91		-4	-1134	324	-36	-36	81	-36	36		-12	12	6	-6	9	-9	12	-12	9	-9											
X.92				486			-54	-54			-18	-18					18	18													
X.93		1560	-312	-40	-40	-24	32	32	12	-8	-8	-8	-8	-4	-4	-16	-16	8	8	8	8	8	8	8	8	8	8	8	8	8	
X.94		-8	-512	640			64				-8																				
X.95		-8	512	-640			-64				8																				
X.96				-108			-54				-27	-12	12																		
X.97				-108			-54				-27	-12	12																		
X.98		3																													
X.99		-4	729		-81	-81																									
X.100				-324	-90	-90	81	-36	-36		12	12																			
X.101		-2	-1232	-224			-80				-8	32	32	-16	-16																
X.102		5	456	240	104	-104	24	-32	32	24	16	-16	-8	8	-4	4															
X.103		5	456	240	-104	104	24	32	-32	24	16	-16	-8	8	4	-4															
X.104			405	-162	45	45	18	18		6	6	21	21	18	18	-6	-6														
X.105			1134	324	-36	36	-36	-36		-12	12	-6	6	18																	

Character table of $H(\text{Fi}_{23})$ (continued)

	2i	5	4	4	2	7	7	7	6	6	3	2	1	4	3	3	4	4	2	2	7	7	5	6	6	6	5	5	5				
3	3	3	3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
5	3	3	3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
7	3	3	3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
11	3	3	3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
13	3	3	3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
	6 ₂₃	6 ₂₄	6 ₂₅	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
2P	3c	3d	3d	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
3P	2f	2f	2f	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
5P	6 ₂₃	6 ₂₄	6 ₂₅	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
7P	6 ₂₃	6 ₂₄	6 ₂₅	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
11P	6 ₂₃	6 ₂₄	6 ₂₅	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
13P	6 ₂₃	6 ₂₄	6 ₂₅	7 _a	8 _a	8 _b	8 _c	8 _d	8 _e	9 _a	9 _b	9 _c	10 _a	10 _b	10 _c	10 _d	10 _e	11 _a	11 _b	12 _a	12 _b	12 _c	12 _d	12 _e	12 _f	12 _g	12 _h	12 _i					
X.88	2	-1	-1							3																							
X.89	3							2	-2		3																						
X.90	3							2	-2				4	2	-2																		
X.91	-3							-2	2				4	-2	2																		
X.92																				1	1												
X.93	-4	-4	-4	2						-3																							
X.94				2						4	-2	-2	-8																				
X.95				2						4	-2	-2	8																				
X.96		3	-3																										-2	2	2		
X.97		3	-3																										-2	2	2		
X.98				-3	-3	-3	1	1					3	-3	-3	-1	-1	1	1														
X.99				-2	-2	2							-4	2	2																		
X.100	3			1	-3	1	-1	-1																					-2	2	6		
X.101									-2	4	1	-2				-2	-2																
X.102				-1					3	3			-5	-1	1	1	-1																
X.103				-1					3	3			-5	1	-1	1	-1																
X.104	-6			1	4	4																											
X.105	-6												5	-1	1	-1	1	1	1														
X.106	6												5	1	-1	-1	1	1	1														
X.107	4	-2	-2		4	-4				3	3																						
X.108		-3	3							-3																							
X.109		-3	3							-3																							
X.110				1						-3			4	2	2																		
X.111				1						-3			4	2	2																		
X.112				1						-3			4	2	2																		
X.113				1						-3			4	2	2																		
X.114				-1	-4	4							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3				

Character table of $H(\text{Fi}_{23})$ (continued)

	2i	5	5	5	4	4	5	5	3	3	1	1	2	2	2	2	5	5	3	2	3	3	3	3	2	2	3		
3	2	2	2	2	2	2	1	1	2	2			1			1			4	4	3	3	3	3	3	3	2	2	3
5																			1										
7													1	1	1														
11													1																
13													1																
	12 _j	12 _k	12 _l	12 _m	12 _n	12 _o	12 _p	12 _q	12 _r	13 _a	13 _b	14 _a	14 _b	14 _c	15 _a	16 _a	16 _b	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i			
2P	6 ₁₈	6 ₁₈	6 ₁₆	6 ₂₀	6 ₁₂	6 ₉	6 ₂₄	6 ₂₄	13 _b	13 _a	7 _a	7 _a	7 _a	15 _a	8 _c	8 _c	9 _a	9 _b	9 _a	9 _a	9 _a	9 _a	9 _a	9 _b	9 _b	9 _a			
3P	4 _e	4 _f	4 _e	4 _b	4 _c	4 _g	4 _d	4 _f	4 _e	13 _a	13 _b	14 _a	14 _b	14 _c	5 _a	16 _a	16 _b	6 ₂	6 ₂	6 ₆	6 ₇	6 ₆	6 ₇	6 ₆	6 ₇	6 ₆			
5P	12 _j	12 _k	12 _l	12 _m	12 _n	12 _o	12 _p	12 _q	12 _r	13 _b	13 _a	14 _a	14 _b	14 _c	3 _a	16 _b	16 _a	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i			
7P	12 _j	12 _k	12 _l	12 _m	12 _n	12 _o	12 _p	12 _q	12 _r	13 _b	13 _a	2 _a	2 _b	2 _c	15 _a	16 _b	16 _a	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i			
11P	12 _j	12 _k	12 _l	12 _m	12 _n	12 _o	12 _p	12 _q	12 _r	13 _b	13 _a	14 _a	14 _b	14 _c	15 _a	16 _b	16 _a	18 _a	18 _b	18 _c	18 _d	18 _e	1						

Character table of $H(\text{Fi}_{23})$ (continued)

2	3	1	3	3	1	2	2	2	2	2	2	4	4	1	1	2	2	2	1
3	2	3	.	.	1	1	1	.	.	1	1	1	1
5	.	.	1	1	1	1	1	1
7	1	1	1	1	1	1	1
11	1	1	1	1	1
13	1	1
18j	18k	20a	20b	21a	22a	22b	22c	22d	22e	22f	24a	24b	26a	26b	30a	30b	30c	42a	
2P	9a	9c	10d	10d	21a	11a	11a	11b	11b	11b	11a	12a	12b	13a	13b	15a	15a	15a	21a
3P	6g	6g	20a	20b	7a	22a	22b	22c	22d	22e	22f	8a	8b	26a	26b	10b	10c	10a	14a
5P	18j	18k	4b	4c	21a	22a	22b	22c	22d	22e	22f	24a	24b	26b	26a	6 ₃	6 ₄	6 ₁	42a
7P	18j	18k	20a	20b	3a	22c	22e	22a	22f	22b	22d	24a	24b	26b	26a	30a	30b	30c	6 ₁
11P	18j	18k	20a	20b	21a	2a	2c	2a	2b	2c	2b	24a	24b	26b	26a	30a	30b	30c	42a
13P	18j	18k	20a	20b	21a	22c	22e	22a	22f	22b	22d	24a	24b	2a	2a	30a	30b	30c	42a
X.88	-1
X.89
X.90	-1	1	1	.	.
X.91	1	-1	1	.	.
X.92	1	1	1	1	1	1	.	.	.	-1	-1
X.93	1	.	.	-1	-1
X.94	-2	.	.	-1	1	1	.	.	-2	-1
X.95	2	.	.	-1	-1	-1	.	.	2	1
X.96	D	C
X.97	C	D
X.98
X.99	.	-1	-1	.	1	1	1	1	1	1	.	.	.	-1	-1
X.100	1	1	1	1	1	1	-2	.	.	.	-1	-1	-1	1	.
X.101	2	1	-2
X.102	1	-1	1	-1	-1	1	1	1	1
X.103	1	.	1	-1	-1	1	-1	1	1	1
X.104	.	.	.	-1	1	1	-1
X.105	.	-1	1	.	-1	-1	-1	-1	-1	-1	1	1	.	.	-1	1	-1	.	.
X.106	.	1	-1	.	-1	1	-1	-1	-1	-1	1	-1	-1	.	.
X.107	-1	1	-1
X.108	-1
X.109	-1
X.110	.	.	.	1	-1	1	-1	1	1	1	-1	-1	1	1	1
X.111	.	.	.	1	1	1	1	-1	1	-1	1	-1	-1	-1	-1
X.112	.	.	.	1	1	-1	1	1	-1	1	-1	1	-1	-1	-1
X.113	.	.	.	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
X.114	.	1	1	-1	1	-1	1	-1	-1	-1	-1	1	.	.	1	1	1	1	-1

where $A = -\zeta(11)_{11}^9 - \zeta(11)_{11}^5 - \zeta(11)_{11}^4 - \zeta(11)_{11}^3 - \zeta(11)_{11} - 1$, $B = 6\zeta(3)_3 + 3$, $C = \zeta(13)_{13}^{11} + \zeta(13)_{13}^8 + \zeta(13)_{13}^7 + \zeta(13)_{13}^6 + \zeta(13)_{13}^5 + \zeta(13)_{13}^2 + 1$, $D = -\zeta(13)_{13}^{11} - \zeta(13)_{13}^8 - \zeta(13)_{13}^7 - \zeta(13)_{13}^6 - \zeta(13)_{13}^5 - \zeta(13)_{13}^2$, $E = -2\zeta(8)_8^3 - 2\zeta(8)_8$.

B.4. Character table of $H(2Fi_{22}) = H_2 = \langle p_2, q_2, h_2 \rangle$

	2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	11	11	13	13	13	13	12
	3	4	4	4	4	4	4	2	2	2	2	1	1	2	2	1	2	2	1	1	1	1	1
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u	
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	6	6	6	6	6	6	6	-2	-2	-2	-2	6	6	-2	-2	6	4	4	2	2	2	2	2
X.4	6	6	6	6	6	6	6	-2	-2	-2	-2	6	6	-2	-2	6	-4	-4	2	2	2	2	2
X.5	10	10	10	10	10	10	10	-6	-6	-6	-6	10	10	-6	-6	10			2	2	2	2	2
X.6	15	15	15	15	15	15	15	7	7	7	7	15	15	7	7	15	5	5	3	3	3	3	3
X.7	15	15	15	15	15	15	15	7	7	7	7	15	15	7	7	15	-5	-5	3	3	3	3	3
X.8	15	15	15	15	15	15	15	-1	-1	-1	-1	15	15	-1	-1	15	-5	-5	-1	-1	-1	-1	-1
X.9	15	15	15	15	15	15	15	-1	-1	-1	-1	15	15	-1	-1	15	5	5	-1	-1	-1	-1	-1
X.10	16	-16	16	-16	16	-16	16	-8	8	-8	8			8	-8		4	-4	4	-4	4	-4	4
X.11	16	-16	16	-16	16	-16	16	-8	8	-8	8			8	-8		4	4	4	4	-4	-4	4
X.12	16	-16	16	-16	16	-16	16	8	-8	8	-8			8	-8		-4	4	4	4	-4	-4	4
X.13	16	-16	16	-16	16	-16	16	8	-8	8	-8			8	-8		4	-4	4	-4	4	-4	4
X.14	20	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20			4	4	4	4	4
X.15	20	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	10	10	4	4	4	4	4
X.16	20	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	-10	-10	4	4	4	4	4
X.17	24	24	24	24	24	24	24	8	8	8	8	24	24	8	8	24	4	4					
X.18	24	24	24	24	24	24	24	8	8	8	8	24	24	8	8	24	-4	-4					
X.19	30	30	30	30	30	30	30	-10	-10	-10	-10	30	30	-10	-10	30	-10	-10	2	2	2	2	2
X.20	30	30	30	30	30	30	30	-10	-10	-10	-10	30	30	-10	-10	30	10	10	2	2	2	2	2
X.21	32	-32	-32	-32	32			16	16	-16	-16							8	-8	8	-8		
X.22	40	40	40	40	40	-40	-40	16	16	16	16	8	8	-16	-16	-8	10	10	4	4	4	4	-4
X.23	40	40	40	40	40	-40	-40	16	16	16	16	8	8	-16	-16	-8	-10	-10	4	4	4	4	-4
X.24	40	40	40	40	40	-40	-40	-16	-16	-16	-16	8	8	16	16	-8	10	10	4	4	4	4	-4
X.25	40	40	40	40	40	-40	-40	-16	-16	-16	-16	8	8	16	16	-8	-10	-10	4	4	4	4	-4
X.26	60	60	60	60	60	60	60	-4	-4	-4	-4	60	60	-4	-4	60	-10	-10	4	4	4	4	4
X.27	60	60	60	60	60	60	60	-4	-4	-4	-4	60	60	-4	-4	60	10	10	4	4	4	4	4
X.28	60	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60			4	4	4	4	4
X.29	64	64	64	64	64	64	64					64	64			64	-16	-16					
X.30	64	64	64	64	64	64	64					64	64			64	16	16					
X.31	80	80	-80	-80	-80	-80	-80	-32	32	32	-32	16	-16					8	-8	-8	8		
X.32	80	80	80	80	80	-80	-80					16	16			-16	20	20	8	8	8	8	-8
X.33	80	80	80	80	80	-80	-80					16	-16			-16	-20	-20	8	8	8	8	-8
X.34	80	80	-80	-80	-80	-80	-80	32	-32	-32	32	16	-16					8	-8	-8	8		
X.35	80	80	80	80	80	80	80	-16	-16	-16	-16	80	80	-16	-16	80							
X.36	81	81	81	81	81	81	81	9	9	9	9	81	81	9	9	81	9	9	-3	-3	-3	-3	-3
X.37	81	81	81	81	81	81	81	9	9	9	9	81	81	9	9	81	-9	-9	-3	-3	-3	-3	-3
X.38	90	90	90	90	90	90	90	-6	-6	-6	-6	90	90	-6	-6	90			-6	-6	-6	-6	-6
X.39	96	-96	96	-96	96	-96	96	-16	16	-16	16			-16	16		16	-16	8	8	-8	-8	-8
X.40	96	-96	96	-96	96	-96	96	-16	16	-16	16			-16	16		-16	16	8	8	-8	-8	-8
X.41	96	-96	96	-96	96	-96	96	-16	16	-16	16			-16	16		-16	16	8	8	-8	-8	-8
X.42	96	-96	96	-96	96	-96	96	-16	16	-16	16			-16	16		-16	16	8	8	-8	-8	-8
X.43	120	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8							
X.44	120	120	120	120	120	120	120	16	16	16	16	24	24	-16	-16	-24	-10	-10	-4	-4	-4	-4	-4
X.45	120	120	120	120	120	120	120	-16	-16	-16	-16	24	24	16	16	-24	-10	-10	-4	-4	-4	-4	-4
X.46	120	120	120	120	120	120	120	-16	-16	-16	-16	24	24	16	16	-24	10	10	-4	-4	-4	-4	-4
X.47	120	120	120	120	120	120	120	16	16	16	16	24	24	-16	-16	-24	10	10	-4	-4	-4	-4	-4
X.48	120	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8							
X.49	120	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8							
X.50	120	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8							
X.51	135	135	135	135	135	135	135	-33	-33	-33	-33	7	7	-33	-33	7	15	15	3	3	3	3	3
X.52	135	135	135	135	135	135	135	-33	-33	-33	-33	7	7	-33	-33	7	-15	-15	3	3	3	3	3
X.53	135	135	135	135	135	135	135	39	39	39	39	7	7	39	39	7	15	15	15	15	15	15	15
X.54	135	135	135	135	135	135	135	39	39	39	39	7	7	39	39	7	-15	-15	15	15	15	15	15
X.55	160	-160	160	-160	160	-160	160	-48	48	-48	48			-48	48			8	8	-8	-8	-8	
X.56	160	-160	160	-160	160	-160	160	-48	48	-48	48			-48	48			8	8	-8	-8	-8	
X.57	160	-160	160	-160	160							32	-32						16	-16	16	16	
X.58	160	-160	160	-160	160			-48	48	48	48							8	-8	8	-8		
X.59	160	-160	160	-160	160			-48	48	48	48							8	-8	8	-8		
X.60	192	-192	192	-192	192			-32	32	32	32							16	-16	16	16		
X.61	216	216	216	216	-216	-216	48	-48	-48	-48	-48	-8	-8	48	48	8	-6	-6	12	12	12	12	-12
X.62	216	216	216	216	-216	-216	48	-48	-48	-48	-48	-8	-8	-48	-48	8	-6	-6	12	12	12	12	-12
X.63	216	216	216	216	-216	-216	48	-48	-48	-48	-48	-8	-8	48	48	8	6	6	12	12	12	12	-12
X.64	216	216	216	216	-216	-216	48	-48	-48	-48	-48	-8	-8	-48	-48	8	6	6	12	12	12	12	-12
X.65	240	240	240	240	-240	-240	-32	32	32	-32	48	48	32	32	-48	-40	-40	8	8	8	8	-8	
X.66	240	240	240	240	-240	-240						32	-32	32	32	48	-48			8	8	8	-8
X.67	240	240	240	240	-240	-240						32	-32	32	32	48	-48			8	8	8	-8
X.68	240	240	240	240	-240	-240	-32	32	32	-32	48	48	-32	-32	-48	40	40	8	8	8	8	-8	
X.69	240	240	240	240	-240	-240	-32	32	32	-32	48	48	32	32	-48	40	40	8	8	8	8	-8	
X.70	24																						

Character table of $H(2\text{Fi}_{22})$ (continued)

	2	3	3	3	5	6	6	6	7	7	7	7	5	5													
	8	8	8	8	9	9	9	8	8	8	8	8	8	8													
	4	4	4	4	3	3	3	3	3	2	2	2	2	2													
	6 ₁	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	
2P	3a	3a	3a	3a	3b	3b	3b	3b	3a	3a	3a	3a	3a	3a	3a	3c	3c	3c	3c	3b	3b	3b	3b	3c	3c	3c	3c
3P	2a	2d	2c	2b	2a	2b	2c	2e	2d	2f	2l	2g	2l	2m	2m	2h	2i	2b	2c	2a	2f	2h	2g	2i	2d	2d	2e
5P	6 ₁	6 ₁	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₄	6 ₁₈	6 ₁₇	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	-3	-3	-3	-3
X.4	-3	-3	-3	-3
X.5	1	1	1	1	4	4	4	4	4	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	-2	-2	
X.6	-3	-3	-3	-3	3	3	3	3	3	1	1	1	1	1	1	1	1	1	1
X.7	-3	-3	-3	-3	3	3	3	3	3	1	1	1	1	1	1	1	1	1	1
X.8	6	6	6	6
X.9	6	6	6	6
X.10	2	-2	2	-2	-4	4	-4	-4	-4	-2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3
X.11	2	-2	2	-2	-4	4	-4	-4	-4	-2	2	2	2	2	2	2	2	2	2	1	-1	-1	-1	-1	-1	-1	-1
X.12	2	-2	2	-2	-4	4	-4	-4	-4	-2	2	2	2	2	2	2	2	2	2	1	-1	-1	-1	-1	-1	-1	-1
X.13	2	-2	2	-2	-4	4	-4	-4	-4	-2	2	2	2	2	2	2	2	2	2	1	-1	-1	-1	-1	-1	-1	-1
X.14	-7	-7	-7	-7	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
X.15	2	2	2	2	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	5	5	5	5	5	5	5	5
X.16	2	2	2	2	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	5	5	5	5	5	5	5	5
X.17	6	6	6	6	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2
X.18	6	6	6	6	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2
X.19	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.20	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.21	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	2	-2	2	-2	2	-2	2	-2
X.22	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	1	1	1	1	1	1	1	1
X.23	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	1	1	1	1	1	1	1	1
X.24	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	1	1	1	1	1	1	1	1
X.25	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	-4	4	1	1	1	1	1	1	1	1
X.26	6	6	6	6	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
X.27	6	6	6	6	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
X.28	-3	-3	-3	-3
X.29	-8	-8	-8	-8	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	4	4	4	4	4	4	4	4
X.30	-8	-8	-8	-8	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	4	4	4	4	4	4	4	4
X.31	8	-8	-8	-8	14	-14	-14	-14	8	2	2	2	2	2	2	2	2
X.32	8	-8	-8	-8	14	-14	-14	-14	8	2	2	2	2	2	2	2	2
X.33	8	-8	-8	-8	14	-14	-14	-14	8	2	2	2	2	2	2	2	2
X.34	8	-8	-8	-8	14	-14	-14	-14	8	2	2	2	2	2	2	2	2
X.35	-10	-10	-10	-10	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	-4	-4	-4	-4	-4	-4	-4	-4
X.36
X.37
X.38	9	9	9	9
X.39	-6	-6	-6	-6	3	-3	-3	-3	-3	-3	-3	-3
X.40	-6	-6	-6	-6	3	-3	-3	-3	-3	-3	-3	-3
X.41	-6	-6	-6	-6	3	-3	-3	-3	-3	-3	-3	-3
X.42	-6	-6	-6	-6	3	-3	-3	-3	-3	-3	-3	-3
X.43	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.44	12	-12	12	-12	3	3	3	3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3	3	3	3
X.45	12	-12	12	-12	3	3	3	3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3	3	3	3
X.46	12	-12	12	-12	3	3	3	3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3	3	3	3
X.47	12	-12	12	-12	3	3	3	3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3	3	3	3
X.48	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.49	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.50	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.51	9	9	9	9	9
X.52	9	9	9	9	9	3	3	3	3	3	3	3
X.53	9	9	9	9	9	3	3	3	3	3	3	3
X.54	9	9	9	9	9	3	3	3	3	3	3	3
X.55	2	-2	2	-2	-16	16	-16	-16	16	6	-6	-6	-6	6	6	6	-6	-2	2	2	2	2	2	2	2	2	2
X.56	2	-2	2	-2	-16	16	-16	-16	16	6	-6	-6	-6	6	6	6	-6	-2	2	2	2	2	2	2	2	2	2
X.57	16	.	-16	-16	8	8	8	8	4	-4	4	-4	4	-4	4	-4
X.58	2	A	-2	2	-16	-16	16	.	-6	B	-6	B	B	B	6	6	2	-2	2	2	2	2	2	2	2	2	2
X.59	2	A	-2	2	-16	-16	16	.	-6	B	-6	B	B	B	6	6	2	-2	2	2	2	2	2	2	2	2	2
X.60	-12	.	12	-12	4	4	-4	-4	-6	6	-6	-8	8	8
X.61	9	9	9	-9	-9	-3	-3	-3	-3	-3	-3	-3
X.62	9	9	9	-9	-9	3	3	3	3	3	3	3
X.63	9	9	9	-9	-9	3	3	3	3	3	3	3
X.64	9	9	9	-9	-9	3	3	3	3	3	3	3
X.65	-12	12	-12	-12	4	-4	4	-4	-4	-4	4	4	3	3	3	3	3	3	3	3	3	3
X.66	24	.	-24	-24	6	-6	-6	.	8	.	8	.	.	.	8	8	-6	-6	6	6	6	6	6	6	6	6	6
X.67	24	.	-24	-24	6	-6	-6	.	8	.	8	.	.	.	8	8	-6	-6	6	6	6	6	6	6	6	6	6
X.68	-12	12	-12	-12	.																						

Character table of $H(2Fi_{22})$ (continued)

	2	8	8	6	6	6	6	7	5	5	5	5	5	5	6	6	6	6	4	4	5	5	5	7	7	7	7				
	3	1	1	2	2	2	2	1	2	2	2	2	2	2	1	1	1	1	2	2	1	1	1	1	1	1	1				
	5																														
	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	8a	8b	8c	8d			
	2P	3P	2j	2k	2m	2o	2p	2l	2n	2f	2m	2l	2i	2m	2g	2l	2h	2q	2r	2s	2t	2o	2p	2u	2x	2v	8a	8b	8c	8d	
	5P	630	631	632	633	634	635	636	637	641	643	640	638	642	639	644	645	646	647	648	649	650	651	652	653	8a	8b	8c	8d		
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
X.2	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1		
X.3	.	.	-2	-2	-2	-2	.	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	1	-1			
X.4	.	.	.	-2	2	2	-2	.	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1			
X.5	4	4	4	2	2	2	2	2	-2		
X.6	1	3	3	1	-1	-1	1	3	-2	-2	-2	-2	-2	-2	-2	2	2	.	1	.	1	1	-1	1		
X.7	3	3	1	1	1	1	3	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	.	-1	.	-1	-1	-1	1		
X.8	.	.	.	-2	-2	-2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	.	-1	-1	-1	-1	1		
X.9	.	.	.	2	2	2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	-1	.	-1	-1	-1	1		
X.10	.	.	.	-2	2	2	2	.	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	.	-1	.	-1	-1	-1	1		
X.11	.	.	.	-2	-2	-2	2	.	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	.	-1	.	-1	-1	-1	1		
X.12	.	.	.	-2	-2	-2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	-1	.	-1	-1	-1	1		
X.13	.	.	.	-2	-2	-2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	-1	.	-1	-1	-1	1		
X.14	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	.	2	.	2	.	4	.		
X.15	-1	-1	-1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1	2	2	2	.		
X.16	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	1	1	1	-2	-2	-2	.	
X.17	3	3	-1	1	1	1	-1	3	3	2	2	2	2	2	2	-2	-2	.	1	.	1	1	1	1	.	
X.18	3	3	-1	-1	-1	-1	-1	3	2	2	2	2	2	2	2	2	2	.	-1	.	-1	-1	-1	-1	.	
X.19	3	3	-1	-1	-1	-1	-1	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	.	
X.20	3	3	-1	1	1	1	-1	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	1	-1	-1	-1	-1	-1	.	
X.21	-2	2	2	2	2	2	2	2	
X.22	-1	-1	-1	1	1	1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1	3	-1	-2	2	2	-2	
X.23	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	
X.24	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2	
X.25	-1	-1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	2	
X.26	-3	-3	-1	-1	-1	-1	-1	-3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	.	
X.27	-3	-3	-1	1	1	1	-1	-3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	
X.28	-2	-2	-2	-2	-2	
X.29	-2	-2	-2	2	2	
X.30	-2	-2	.	-2	-2	.	-2	-2	-2	
X.31	-2	-2	.	-2	-2	.	-2	-2	-2	
X.32	4	4	.	2	2	.	.	4	2	2	2	2	2	2	2	.	-2	-2	-2	-2	-2	-2	.	
X.33	4	4	.	-2	-2	.	-4	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	.	
X.34	-2	2	2	2	-2	-2	-2	-2	-2	-2	
X.35	2	2	2	.	.	.	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
X.36	-3	-3	-3	-1	.	
X.37	3	3	3	-1	2	
X.38	
X.39	.	.	.	4	4	-4	-4	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	.	
X.40	.	.	.	4	4	-4	-4	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	.	
X.41	.	.	.	4	4	-4	-4	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	.	
X.42	.	.	.	4	4	-4	-4	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	.	
X.43	-2	-2	-2	-2
X.44	3	3	-1	-1	-1	-1	-1	-3	1	-1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	-2	2	2	2	.	
X.45	3	3	-1	-1	-1	-1	-1	-3	-1	1	1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	
X.46	3	3	1	1	1	1	1	-3	-1	1	1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	2	
X.47	-3	-3	-1	1	1	1	-1	-3	-1	-1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	2	
X.48	-2	-2	-2	2	2	
X.49	-2	-2	-2	2	2	
X.50	-2	-2	-2	2	2	
X.51	1	1	1	3	3	3	3	1	1	.	-1	-1	-1	-1	1	
X.52	1	1	3	-3	-3	3	3	1	-1	.	1	1	1	1	1	
X.53	1	1	3	3	3	3	1	1	.	-1	-1	-1	-1	1	
X.54	1	1	3	-3	-3	3	1	-1	.	1	1	1	1	1	
X.55	2	2	-2	-2	-2	-2	.	2	.	-2		
X.56	2	2	-2	-2	-2	-2	.	-2	.	2		
X.57	8	-8	4	-4	-4	4	4	4	4	
X.58	B	B	B	B	B	B	B	2	-2	2	-2	-2	-2	
X.59	B	B	B	B	B	B	B	2	-2	2	-2	-2	-2	
X.60	-2	2	-2	2	-2	2	2	2	
X.61	1	1	3	-3	-3	3	-1	1	.	2	-2	-2	-2	.	
X.62	1	1	-3	-3	-3	-3	-1	-1	.	-2	-2	-2	-2	-2	.	
X.63	1	1	-3	3	3	3	-1	1	.	2	2	2	2	2	.	
X.64	1	1	3	3	3	3	-1	-1	.	-2	-2	-2	-2	-2	.	
X.65	.	.	.	2	2	2	2	.	1	-1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	.	1		
X.66	6	-6	-2	-2	2	2	2	2					

Character table of $H(2F_{i_{22}})$ (continued)

	2	7	7	7	6	6	6	6	6	6	3	4	4	4	3	3	3	3	7	7	5	6	6	6	6	6	6				
	3	2	1	1	1	1	1	1	1	2	2	3	2	2	2	2	1	1	1			
	5	8e	8f	8g	8h	8i	8j	8k	8l	8m	8n	9a	10a	10b	10c	10d	10e	10f	10g	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈	12 ₉			
2P	4 ₃₀	4 ₂₀	4 ₂₀	4 ₁₁	4 ₁₁	4 ₁₆	4 ₁₉	4 ₂₇	4 ₂₇	4 ₁₆	9a	5a	5a	5a	5a	5a	5a	5a	6 ₈	6 ₈	6 ₄	6 ₉	6 ₉	6 ₈	6 ₈	6 ₃₀	6 ₃₀	6 ₃₀			
3P	8e	8f	8g	8h	8i	8j	8k	8l	8m	8n	3a	10a	10b	10c	10d	10e	10f	10g	4 ₁	4 ₁	4 ₁	4 ₃	4 ₂	4 ₁	4 ₅	4 ₆	4 ₇	4 ₇			
5P	8e	8f	8g	8h	8i	8j	8k	8l	8m	8n	9a	2b	2a	2c	2o	2d	2p	2e	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈	12 ₉	12 ₉			
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
X.2	-1	1	-1	-1	-1	-1	-1	1	1	-1	1	1	1	1	-1	1	-1	1	1	1	1	1	1	1	-1	-1	1	1			
X.3	-2	.	2	2	1	1	1	-1	1	-1	1	.	.	-3	-2	-2	.	-2	-2		
X.4	2	.	-2	-2	1	1	1	1	1	1	1	.	.	-3	-2	2	.	-2	-2		
X.5	-2	-2	-2	1	4	4	1	.	4	.		
X.6	3	1	-1	-1	-1	-1	-1	-1	1	-1	3	3	3	-3	-1	-1	3	1	1	1			
X.7	-3	1	1	1	1	1	1	1	1	1	3	3	3	-3	1	1	3	-1	1	1			
X.8	-1	-1	-1	1	1	1	-1	-1	-1	1	6	-2	-2	.	.	2	2		
X.9	1	-1	1	-1	-1	-1	1	-1	-1	-1	6	2	2	.	.	2	2		
X.10	-2	1	1	-1	-1	-1	1	1	-1	-2	-2	.	.	2		
X.11	-2	1	1	-1	-1	1	1	-1	-1	-2	-2	.	.	2		
X.12	.	.	.	-2	2	1	1	-1	-1	1	1	-1	-1	-2	-2	.	.	2		
X.13	.	.	.	2	-2	1	1	-1	-1	-1	-1	1	1	-2	-2	.	.	2		
X.14	-1	2	2	.	.	.	-7	2	.	.	-2	-2		
X.15	2	.	2	2	-1	-1	1	1	1	1	-1	-1	1	1	1		
X.16	-2	.	-2	-2	.	.	-1	-1	-1	2	2	2	-1	-1	-1	1	1	1		
X.17	-1	-1	-1	-1	-1	-1	-1	3	3	6	1	1	3	3	1	1	3	1	1		
X.18	-1	-1	-1	1	-1	1	-1	3	3	6	-1	-1	3	3	-1	-1	3	-1	-1		
X.19	4	3	3	3	-1	-1	3	3	-1	-1	3	-1	-1		
X.20	-4	3	3	3	1	1	3	3	1	1	3	1	-1	-1	
X.21	2	-2	-2	2	2	
X.22	.	2	1	3	3	.	-1	-1	-3	-1	1	1	1	1		
X.23	.	2	1	3	3	.	1	1	-3	1	1	1	1	1		
X.24	.	-2	1	3	3	.	-1	-1	-3	-3	-1	-1	-1	-1		
X.25	.	-2	1	3	3	.	1	1	-3	3	-1	-1	-1	-1		
X.26	-2	.	2	-3	-3	.	6	-1	-1	-3	1	-1	-1	-1		
X.27	2	.	-2	-2	-3	-3	.	6	1	1	-3	-1	-1	-1	-1		
X.28	-3		
X.29	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	.	.	-2	-2		
X.30	1	-1	-1	-1	1	-1	1	-1	-2	-2	.	-8	-2	-2	-2		
X.31	-1	-1	-1	-1	1	-1	1	-1	-2	-2	.	.	.	-2	-2		
X.32	-2	2	.	2	6	-6	.	.	.	-2	-2	.	.	2	2		
X.33	-1	2	2	.	.	-2	-2		
X.34	2	-2	2	6	-6	2	-2		
X.35	-1	2	2	-10	.	.	.	2	.	.	2	2		
X.36	3	-1	-1	1	1	1	-1	-1	-1	1	1	1	1	-1	1	-1	1	1		
X.37	-3	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	1	1	1	1		
X.38	2	.	2	2	2	9	
X.39	1	-1	-1	1	-1	-1	1	4	-4		
X.40	1	-1	-1	-1	-1	1	1	-4	4		
X.41	1	-1	-1	-1	1	1	-1	-4	-4		
X.42	1	-1	-1	1	1	-1	-1	-4	4		
X.43	.	.	.	2	2	-2	.	.	-2	2	2	.	-1	.	.	.	2	2	.	.		
X.44	-2	3	3	.	.	1	1	-3	1	1	1	1		
X.45	3	3	.	.	1	1	-3	-1	-1	-1	-1		
X.46	3	3	.	.	1	1	-3	1	1	-1	-1		
X.47	3	3	.	.	-1	-1	-3	1	1	-1	-1		
X.48	-2	2	2	.	.	-1	-1	-3	-1	1	1	1		
X.49	.	.	2	2	-2	.	.	.	-2	2	2	.	-1	.	.	-2	-2	.	.	.		
X.50	.	.	2	2	-2	.	.	.	-2	2	2	.	-1	.	.	-2	-2	.	.	.		
X.51	1	1	-3	-1	-1	1	1	1	-1	1	1	1	-1	1	1	-1	1	.	-3	-3	.	3	3	-3	3	1	-1	-1	-1		
X.52	-1	1	3	1	1	1	-1	-1	-1	1	1	1	1	1	1	1	1	.	-3	-3	.	-3	-3	-3	-1	-1	-1	-1	-1		
X.53	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	1	1	1	1	1	.	-3	-3	.	3	3	-3	3	1	-1	-1	-1		
X.54	1	-1	1	-1	-1	-1	1	-1	-1	-1	-3	-3	.	-3	-3	-3	-1	-1	-1	-1	-1		
X.55	1	
X.56	-2	
X.57	1	
X.58	1	
X.59	1	
X.60	-2	-2	2	
X.61	.	2	1	1	1	1	-1	-1	-1	-1	-3	-3	.	3	3	3	1	1	1	1	1		
X.62	.	-2	1	1	1	1	-1	-1	-1	-1	-3	-3	.	-3	-3	-3	-1	-1	-1	-1	-1		
X.63	.	-2	1	1	1	1	-1	-1	-1	-1	-3	-3	.	-3	-3	-3	1	-1	-1	-1	-1		
X.64	.	2	1	1	1	1	-1	-1	-1	-1	-3	-3	.	-3	-3	3	-1	1	1	1	1		
X.65	-2	-2	.	.	-2	-2	-2	
X.66	-2	2	6	-6	-2	-2	
X.67	2	-2	6	-6	-2	-2	
X.68	2	2	
X.69	2	2	-2	-2
X.7																															

Character table of $H(2\text{Fi}_{22})$ (continued)

	2	4	5	5	3	3	3	3	3	3	3	3	3	4	4
3	1	.	.	2	2	2	2	2	2	2	2	2	2	1	1
5
	12 ₃₁	16a	16b	18a	18b	18c	18d	18e	18f	18g	20a	20b	24a	24b	
2F	6 ₄₅	8c	8c	9a	9a	9a	9a	9a	9a	9a	10a	10a	12 ₁	12 ₂	
3F	4 ₂₆	16a	16b	6 ₂	6 ₁	6 ₃	6 ₆	6 ₅	6 ₇	6 ₄	20a	20b	8a	8b	
5F	12 ₃₁	16b	16a	18c	18e	18a	18d	18b	18f	18g	4 ₂	4 ₃	24a	24b	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	-1	-1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	
X.3	1	-1	-1	.	.	
X.4	-1	1	1	.	.	
X.5	.	.	.	1	1	1	1	1	1	1	
X.6	.	-1	-1	1	1	
X.7	.	1	1	-1	-1	
X.8	-1	1	1	
X.9	1	-1	-1	
X.10	-1	.	.	-1	1	-1	-1	1	1	-1	-1	1	.	.	
X.11	1	.	.	-1	1	-1	-1	1	1	-1	1	-1	.	.	
X.12	-1	.	.	1	-1	1	-1	-1	1	-1	-1	1	.	.	
X.13	1	.	.	1	-1	1	-1	-1	1	-1	1	-1	.	.	
X.14	.	.	.	-1	-1	-1	-1	-1	-1	-1	
X.15	-1	.	.	-1	-1	-1	-1	-1	-1	-1	.	.	-1	-1	
X.16	1	.	.	-1	-1	-1	-1	-1	-1	-1	.	.	1	1	
X.17	-1	-1	1	1	
X.18	1	1	-1	-1	
X.19	1	1	1	
X.20	-1	1	1	
X.21	2	.	-2	-2	
X.22	1	.	.	-1	-1	-1	1	-1	1	1	.	.	1	-1	
X.23	-1	.	.	-1	-1	-1	1	-1	1	1	.	.	-1	1	
X.24	-1	.	.	-1	-1	-1	1	-1	1	1	.	.	-1	1	
X.25	1	.	.	-1	-1	-1	1	-1	1	1	.	.	1	-1	
X.26	-1	1	1	
X.27	1	-1	-1	
X.28	
X.29	.	.	.	1	1	1	1	1	1	1	-1	-1	.	.	
X.30	.	.	.	1	1	1	1	1	1	1	1	1	.	.	
X.31	-2	.	-2	2	
X.32	.	.	.	1	1	1	-1	1	-1	-1	
X.33	.	.	.	1	1	1	-1	1	-1	-1	
X.34	-2	.	-2	2	
X.35	.	.	.	-1	-1	-1	-1	-1	-1	-1	
X.36	.	1	1	-1	-1	.	.	
X.37	.	-1	-1	1	1	.	.	
X.38	
X.39	1	-1	1	.	.	
X.40	-1	1	-1	.	.	
X.41	1	-1	1	.	.	
X.42	-1	1	-1	.	.	
X.43	
X.44	-1	1	-1	
X.45	1	-1	1	
X.46	-1	1	-1	
X.47	1	-1	1	
X.48	
X.49	
X.50	
X.51	.	1	1	-1	-1	
X.52	.	-1	-1	1	1	
X.53	.	-1	-1	-1	-1	
X.54	.	1	1	1	1	
X.55	.	.	.	-1	1	-1	-1	1	1	-1	
X.56	.	.	.	1	-1	1	-1	-1	1	-1	
X.57	2	.	2	-2	
X.58	.	.	.	D	\bar{D}	\bar{D}	1	D	-1	-1	
X.59	.	.	.	\bar{D}	D	D	1	\bar{D}	-1	-1	
X.60	
X.61	1	1	-1	1	
X.62	1	1	1	-1	
X.63	-1	-1	-1	1	
X.64	-1	-1	1	-1	
X.65	-1	
X.66	
X.67	
X.68	-1	
X.69	1	
X.70	
X.71	
X.72	1	
X.73	-1	
X.74	-1	
X.75	1	
X.76	
X.77	
X.78	1	
X.79	
X.80	.	E	\bar{E}	
X.81	.	\bar{E}	E	
X.82	1	1	
X.83	-1	-1	
X.84	.	.	.	1	1	1	-1	1	-1	-1	
X.85	1	.	.	1	-1	1	1	-1	-1	1	
X.86	-1	.	.	1	-1	1	1	-1	-1	1	
X.87	.	.	.	1	1	1	-1	1	-1	-1	
X.88	.	.	.	1	-1	1	1	-1	-1	1	
X.89	.	.	.	-1	1	-1	1	1	-1	1	
X.90	1	.	.	-1	1	-1	1	1	-1	1	
X.91	-1	.	.	-1	1	-1	1	1	-1	1	
X.92	.	.	.	D	\bar{D}	\bar{D}	-1	D	1	1	
X.93	.	.	.	\bar{D}	D	D	-1	\bar{D}	1	1	
X.94	.	.	.	D	D	\bar{D}	1	\bar{D}	1	-1	
X.95	.	.	.	\bar{D}	\bar{D}	D	1	D	1	-1	

Character table of $H(2F_{122})$ (continued)

	2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	11	11	13	13
	3	4	4	4	4	4	4	2	2	2	2	1	1	2	2	1	2	2	1	1
	5	1	1	1	1	1	1										1	1		
		1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
2P		1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P		1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
5P		1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
X.96	320	320	-320	-320	.	.	-64	64	64	-64	64	-64	64
X.97	320	320	-320	-320	.	.	-64	64	64	-64	64	-64	64
X.98	384	-384	384	-384	384	-384	-64	64	-64	64	.	.	.	64	-64	.	16	-16	.	.
X.99	384	-384	384	-384	-384	384	64	-64	64	-64	.	.	.	64	-64	.	-16	16	.	.
X.100	384	-384	384	-384	384	-384	-64	64	-64	64	.	.	.	64	-64	.	-16	16	.	.
X.101	384	-384	384	-384	-384	384	64	-64	64	-64	.	.	.	64	-64	.	16	-16	.	.
X.102	405	405	405	405	405	405	45	45	45	45	21	21	45	45	21	45	45	9	9	9
X.103	405	405	405	405	405	405	-27	-27	-27	-27	21	21	-27	-27	21	45	45	-3	-3	-3
X.104	405	405	405	405	405	405	-27	-27	-27	-27	21	21	-27	-27	21	45	45	-3	-3	-3
X.105	405	405	405	405	405	405	45	45	45	45	21	21	45	45	21	45	45	9	9	9
X.106	432	432	-432	-432	.	.	96	-96	-96	96	-16	16	24	-24
X.107	432	432	-432	-432	.	.	-96	96	96	-96	16	-16	24	-24
X.108	480	480	480	480	-480	-480	-32	-32	-32	-32	96	96	32	32	-96	-40	-40	.	.	.
X.109	480	480	-480	-480	.	.	64	-64	-64	64	96	-96	16	-16
X.110	480	480	480	480	-480	-480	96	96	-16	16
X.111	480	-480	480	-480	480	-480	80	-80	80	-80	.	.	-80	80	.	-40	40	8	8	8
X.112	480	-480	480	-480	480	-480	80	80	-80	80	.	.	-80	80	.	-40	40	8	8	8
X.113	480	480	480	480	-480	-480	32	32	32	32	96	96	-32	-32	-96	-40	-40	.	.	.
X.114	480	-480	480	-480	480	-480	80	80	-80	80	.	.	-80	80	.	-40	-40	8	8	8
X.115	480	480	-480	-480	.	.	-64	64	64	-64	96	-96	16	-16
X.116	480	-480	480	-480	480	-480	80	-80	80	-80	.	.	-80	80	.	40	-40	8	8	8
X.117	480	480	480	480	-480	-480	-32	-32	-32	-32	96	96	32	32	-96	40	40	.	.	.
X.118	480	-480	480	-480	480	-480	112	112	-112	-112	24	-24
X.119	480	480	480	480	-480	-480	32	32	32	32	96	96	-32	-32	-96	40	40	.	.	.
X.120	480	-480	480	-480	480	-480	-16	-16	16	16	-8	8
X.121	480	480	-480	-480	96	-96	-16	16
X.122	480	480	-480	-480	96	-96	-16	16
X.123	540	540	540	540	540	540	60	60	60	60	28	28	60	60	28	-30	-30	-12	-12	-12
X.124	540	540	540	540	540	540	-84	-84	-84	-84	28	28	-84	-84	28	30	30	12	12	12
X.125	540	540	540	540	540	540	-84	-84	-84	-84	28	28	-84	-84	28	-30	-30	12	12	12
X.126	540	540	540	540	540	540	60	60	60	60	28	28	60	60	28	30	30	-12	-12	-12
X.127	640	640	640	640	-640	-640	128	128	.	.	-128
X.128	640	-640	-640	-640	640	640	64	64	-64	-64	.	.	128	-128	32	-32
X.129	640	640	-640	-640	640	640	128	-128
X.130	640	640	-640	-640	640	640	128	-128
X.131	720	720	720	720	720	720	-48	-48	-48	-48	-48	-48	-48	-48	-48
X.132	720	720	720	720	720	720	-48	-48	-48	-48	-48	-48	-48	-48	-48
X.133	720	720	720	720	720	720	-48	-48	-48	-48	-48	-48	-48	-48	-48
X.134	720	720	720	720	720	720	-48	-48	-48	-48	-48	-48	-48	-48	-48
X.135	768	-768	-768	768	.	.	128	128	-128	-128
X.136	810	810	810	810	810	810	90	90	90	90	42	42	90	90	42	.	.	18	18	18
X.137	810	810	810	810	810	810	18	18	18	18	42	42	18	18	42	.	.	-18	-18	-18
X.138	810	810	810	810	810	810	-54	-54	-54	-54	42	42	-54	-54	42	.	.	-6	-6	-6
X.139	810	810	810	810	810	810	18	18	18	18	42	42	18	18	42	.	.	-18	-18	-18
X.140	864	864	864	864	-864	-864	96	96	96	96	-32	-32	-96	-96	32	24	24	.	.	.
X.141	864	864	864	864	-864	-864	96	96	96	96	-32	-32	-96	-96	32	24	24	.	.	.
X.142	864	864	864	864	-864	-864	96	96	96	96	-32	-32	-96	-96	32	24	24	.	.	.
X.143	864	864	864	864	-864	-864	96	96	96	96	-32	-32	-96	-96	32	24	24	.	.	.
X.144	960	960	-960	-960	.	.	-64	64	64	-64	192	-192
X.145	960	960	960	960	960	960	-64	-64	.	.	-64
X.146	960	960	-960	-960	.	.	64	-64	-64	64	192	-192
X.147	960	-960	960	-960	960	960	-96	96	-96	96	.	.	96	-96	.	.	.	16	16	16
X.148	960	-960	-960	-960	960	960	-160	-160	160	160	16	-16	-16
X.149	960	960	960	960	960	960	-64	-64	.	.	-64
X.150	960	-960	960	-960	-960	-960	96	-96	96	-96	.	.	96	-96	.	.	.	16	16	16
X.151	960	-960	960	-960	-960	-960	32	32	-32	32	.	.	-32	32	.	-40	40	16	16	16
X.152	960	-960	960	-960	-960	-960	32	32	-32	32	.	.	-32	32	.	-40	40	16	16	16
X.153	960	-960	960	-960	-960	-960	32	32	-32	32	.	.	-32	32	.	-40	40	16	16	16
X.154	960	-960	960	-960	-960	-960	32	32	-32	32	.	.	-32	32	.	-40	40	16	16	16
X.155	960	-960	960	960	.	.	96	96	-96	-96	16	-16	-16
X.156	960	-960	960	960	.	.	96	96	-96	-96	16	-16	-16
X.157	1024	-1024	1024	-1024	1024	-1024	-64	64	.	.	.
X.158	1024	-1024	1024	-1024	-1024	1024	-64	64	.	.	.
X.159	1024	-1024	1024	-1024	-1024	1024	-64	64	.	.	.
X.160	1024	-1024	1024	-1024	1024	-1024	64	-64	.	.	.
X.161	1080	1080	1080	1080	-1080	-1080	48	48	48	48	-40	-40	-48	-48	40	30	30	12	12	12
X.162	1080	1080	1080	1080	-1080	-1080	-48	-48	-48	-48	-40	-40	48	48	40	30	30	12	12	12
X.163	1080	1080	1080	1080	1080	1080	-24	-24	-24	-24	56	56	-24	-24	56	-60	-60	.	.	.
X.164	1080	1080	1080	1080	-1080	-1080	-48	-48	-48	-48	-40	-40	48	48	40	-30	-30	12	12	12
X.165	1080	1080	1080	1080	1080	1080	-24	-24	-24	-24	56	56	-24	-24	56	-60	-60	.	.	.
X.166	1080	1080	1080	1080	-1080	-1080	48	48	48	48	-40	-40	-48	-48	40	-30	-30	12	12	12
X.167	1280	-1280	1280	-1280	1280	-1280	128	-128	-128	128	.	.	-128	128
X.168	1280	-1280	1280	-1280	1280	-1280	128	-128	-128	128	.	.	-128	128
X.169	1280	-1280	1280	-1280	1280	-1280	128	-128	-128	128	.	.	-128	128
X.170	1280	-1280	1280	-1280	1280	-1280	128	-128	-128	128	.	.	-128	128
X.171	1296	1296	1296	-1296	-1296	-1296	-48	-48	.	.	48	36	36	-24	-24	-24
X.172	1296	-1296	1296	-1296	1296	-1296	-72	72	-72											

Character table of $H(2\text{Fi}_{22})$ (continued)

	2	3	4	8	8	8	8	8	8	8	8	9	9	9	8	8	8	8	8	8	8	8	8	6	6	6
	1	1	1	4	4	4	4	4	4	4	4	3	3	3	3	3	2	2	2	2	2	2	2	3	3	3
	4 ₄₁	5 _a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	
	2 _P	2 _q	3 _a	3 _b	3 _c	3 _a	3 _a	3 _a	3 _a	3 _a	3 _b	3 _b	3 _b	3 _b	3 _a	3 _a	3 _a	3 _a	3 _a	3 _a	3 _a	3 _a	3 _c	3 _c	3 _c	
	3 _F	4 ₄₁	5 _a	2 _d	2 _e	2 _e	2 _a	2 _d	2 _c	2 _b	2 _a	2 _b	2 _c	2 _e	2 _d	2 _f	2 _l	2 _g	2 _l	2 _m	2 _m	2 _h	2 _i	2 _b	2 _c	
	5 _P	4 ₄₁	1 _a	6 ₅	6 ₃	6 ₂	6 ₄	6 ₁	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₆	6 ₁₅	6 ₁₄	6 ₁₈	6 ₁₇	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	
X.96				\bar{F}	\bar{F}	\bar{F}	-4	\bar{F}	4	4	14	-14	-14			-4	G	4	G	G	G	4	-4	4	-4	
X.97				\bar{F}	\bar{F}	\bar{F}	-4	\bar{F}	4	4	14	-14	-14			-4	G	4	G	G	G	4	-4	4	-4	
X.98				-1	-12	12	12	-12	12	-12	-12	12	-12	-12	-12	12	-4	4	4	4	4	-4	-4	-4	4	-4
X.99				-1	12	-12	-12	12	12	12	-12	-12	-12	12	-12	12	-4	4	4	4	4	-4	-4	-4	4	-4
X.100				-1	-12	12	12	-12	12	-12	-12	12	-12	-12	12	-4	4	4	4	4	4	-4	-4	-4	4	-4
X.101				-1	12	-12	-12	12	12	12	-12	-12	-12	12	-12	12	-4	4	4	4	4	-4	-4	-4	4	-4
X.102				-1																						
X.103				1																						
X.104				1																						
X.105				-1																						
X.106												18	-18	-18												
X.107				2								18	-18	-18												
X.108					-12	-12	-12	-12	12	12	-3	-3	-3	3	3	4	-4	4	-4	-4	-4	4	4	4	4	4
X.109																										
X.110					24	24	24	24	-24	-24	-24	-24	-24													
X.111					-6	6	6	6	-6	-6	6	-6	-6	-12	12	-12	-2	-2	-2	-2	2	2	-2	-2	3	-3
X.112					6	-6	-6	-6	6	6	-6	-6	-6	12	-12	12	-2	-2	-2	-2	2	2	-2	-2	3	-3
X.113					-12	-12	-12	-12	12	12	-3	-3	-3	3	3	4	-4	4	-4	-4	-4	4	4	4	4	4
X.114					6	-6	-6	-6	6	6	-6	-6	-6	12	-12	12	-2	-2	-2	-2	2	2	-2	-2	3	-3
X.115																										
X.116					-6	6	6	6	-6	-6	6	-6	-6	-12	12	-12	-2	-2	-2	-2	2	2	-2	-2	3	-3
X.117					-12	-12	-12	-12	12	12	-3	-3	-3	3	3	4	-4	4	-4	-4	-4	4	4	4	4	4
X.118																										
X.119					-12	-12	-12	-12	12	12	-3	-3	-3	3	3	4	-4	4	-4	-4	-4	4	4	4	4	4
X.120																										
X.121																										
X.122																										
X.123												9	9	9	9	9										
X.124												9	9	9	9	9										
X.125												9	9	9	9	9										
X.126												9	9	9	9	9										
X.127					8	8	8	-8	8	-8	-8	-8	-8	8	8	8										
X.128												8	8	8	8	8										
X.129												8	8	8	8	8										
X.130												8	8	8	8	8										
X.131					-9	-9	-9	-9	-9	-9	-9															
X.132					-9	-9	-9	-9	-9	-9	-9															
X.133					-9	-9	-9	-9	-9	-9	-9															
X.134					-9	-9	-9	-9	-9	-9	-9															
X.135					-2																					
X.136																										
X.137																										
X.138																										
X.139																										
X.140																										
X.141																										
X.142																										
X.143																										
X.144																										
X.145					24	24	24	24	24	24	-6	-6	-6	-6	-6	-6										
X.146																										
X.147					6	-6	-6	-6	6	-6	6															
X.148																										
X.149					24	24	24	24	24	24	-6	-6	-6	-6	-6	-6										
X.150					-6	6	6	6	-6	-6	6															
X.151					12	-12	-12	-12	12	12	-12	-12	-12	12	-12	12	-4	4	-4	-4	-4	4	4	-4	-4	4
X.152					-12	12	12	12	-12	-12	12	-12	-12	12	-12	12	-4	4	-4	-4	-4	4	4	-4	-4	4
X.153					12	-12	-12	-12	12	12	-12	-12	-12	12	-12	12	-4	4	-4	-4	-4	4	4	-4	-4	4
X.154					-12	12	12	12	-12	-12	12	-12	-12	12	-12	12	-4	4	-4	-4	-4	4	4	-4	-4	4
X.155																										
X.156																										
X.157					-1	16	-16	-16	-16	16	-16	16	8	-8	8	8	-8									
X.158					-1	16	16	16	-16	-16	16	8	-8	8	-8	8	-8									
X.159					-1	16	16	16	-16	-16	16	8	-8	8												

Character table of $H(2\text{Fi}_{22})$ (continued)

	2 3	7 2	7 2	7 2	7 2	5 3	5 3	8 1	8 1	6 2	6 2	6 2	6 2	7 1	5 2	5 2	5 2	5 2	5 2	5 2	5 2	5 2	5 2	6 1	6 1	6 1	6 1	4 2	4 2
	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650		
	2f	2h	2g	2i	2d	2e	2j	2k	2m	2o	2p	2l	2n	2f	2m	2l	2i	2m	2g	2l	2h	2q	2r	2s	2t	2o	2p		
5P	624	625	626	627	628	629	630	631	632	633	634	635	636	637	641	643	640	638	642	639	644	645	646	647	648	649	650		
X.96	2	-2	-2	2											2	B	B	2	B	-2	B	-2							
X.97	2	-2	-2	2											2	B	B	2	B	-2	B	-2							
X.98	2	-2	-2	2											2	B	B	2	B	-2	B	-2							
X.99	-2	-2	2	2											-2	B	B	-2	B	2	B	2							
X.100	2	-2	2	-2											2	B	B	-2	B	2	B	2							
X.101	-2	-2	2	2											-2	B	B	2	B	-2	B	-2							
X.102																													
X.103																													
X.104																													
X.105																													
X.106	6	-6	-6	6											2	-2													
X.107	-6	6	6	-6											-2	2													
X.108	1	1	1	1											-3	-3	-1	-1	-1	-1	3	-2	2	2	-2	2	-2	2	2
X.109	4	-4	-4	4											-2														
X.110																													
X.111	2	-2	-2	2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.112	-2	-2	2	2											1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
X.113	-1	-1	1	1											3	2	-2	-2	2	2	-2	-2	2	2	-2	-2	2	2	-2
X.114	-2	-2	2	2											1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
X.115	-4	4	4	-4											2														
X.116	2	-2	-2	2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.117	1	1	1	1											-3	-3	-1	-1	1	1	-1	-1	3	-2	2	2	-2	2	-2
X.118	4	-4	4	-4											4														
X.119	-1	-1	1	1											-3	-3	1	1	-1	-1	3	-2	-2	2	2	-2	-2	2	-2
X.120	8	-8	8	-8											2														
X.121																					B	B	B	B	2	-2	-2	2	
X.122																					B	B	B	B	2	-2	-2	2	
X.123	-3	-3	-3	-3											1	1	-3	-3	3	3	-1								
X.124	-3	-3	-3	-3											1	1	-3	-3	3	3	-1								
X.125	-3	-3	-3	-3											1	1	-3	-3	3	3	-1								
X.126	-3	-3	-3	-3											1	1	-3	-3	3	3	-1								
X.127																													
X.128	4	-4	4	-4											-2						2		-2	2	2	-2	2	-2	
X.129																													
X.130																													
X.131																													
X.132																													
X.133																													
X.134																													
X.135	-4	4	-4	4											-4						4		-4	4					
X.136																													
X.137																													
X.138																													
X.139																													
X.140	-3	-3	-3	-3											1	1	3	3	3	3	-1								
X.141	3	3	3	3											1	1	-3	-3	-3	-3	1								
X.142	-3	-3	-3	-3											1	1	3	3	3	3	-1								
X.143	3	3	3	3											1	1	-3	-3	-3	-3	1								
X.144	2	-2	-2	2											-6	6					-4		-4	4	4				
X.145	2	-2	-2	2											-6	6					-4		-4	4	4				
X.146	-2	2	2	-2											2	2					4		4	-4	-4				
X.147	-4	4	-4	4											-6	6					4		4	-4	-4				
X.148	-4	4	-4	4											-6	6					4		4	-4	-4				
X.149															2	2					2		-2	2	2	-2	2	-2	
X.150															2	2					2		-2	2	2	-2	2	-2	
X.151	-2	-2	2	2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.152	-2	-2	2	2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.153	-2	-2	2	2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.154	2	2	-2	-2											-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
X.155																					B	B	B	B	-2	2	-2	2	
X.156																					B	B	B	B	-2	2	-2	2	
X.157																													
X.158																													
X.159																													
X.160																													
X.161	3	3	3	3											-1	-1	3	3	3	3	-1								
X.162	-3	-3	-3	-3											-1	-1	3	3	3	3	-1								
X.163	3	3	3	3											-1	-1	3	3	3	3	-1								
X.164	-3	-3	-3	-3											-1	-1	3	3	3	3	-1								
X.165	3	3	3	3											-1	-1	3	3	3	3	-1								
X.166	3	3	3	3											-1	-1	3	3	3	3	-1								
X.167	-4	-4	4	4											-4						4		4	-4	-4				
X.168	4	4	-4	-4											-4						4		4	-4	-4				
X.169	4	-4	4	-4											-2	B	B	2	B	-2	B	2							
X.170	4	-4	4	-4											-2	B	B	2	B	-2	B	2							
X.171																													
X.172																													

Character table of $H(2\text{Fi}_{22})$ (continued)

	2	5	5	5	4	4	4	4	5	5	3	3	3	3	3	3	3	3	4	4
	3	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1
	5																			
	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	12 ₂₉	12 ₃₀	12 ₃₁	16 _a	16 _b	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	20 _a	20 _b	24 _a	24 _b
	6 ₉	6 ₁₅	6 ₁₃	6 ₄₆	6 ₄₅	6 ₄₆	6 ₄₅	8 _c	8 _c	9 _a	9 _a	9 _a	9 _a	9 _a	9 _a	9 _a	10 _a	10 _a	12 ₁	12 ₂
	4 ₁₇	4 ₁₆	4 ₁₈	4 ₂₁	4 ₂₂	4 ₂₄	4 ₂₆	16 _a	16 _b	6 ₂	6 ₁	6 ₃	6 ₆	6 ₅	6 ₇	6 ₄	20 _a	20 _b	8 _a	8 _b
	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	12 ₂₉	12 ₃₀	12 ₃₁	16 _a	16 _b	18 _c	18 _e	18 _a	18 _d	18 _b	18 _f	18 _g	4 ₂	4 ₃	24 _a	24 _b
X.96	D	D	D	1	D	1	-1
X.97	D	D	D	1	D	1	-1
X.98	1	-1	.	.
X.99	1	-1	.	.
X.100	-1	1	.	.
X.101	-1	1	.	.
X.102	1	1
X.103	-1	-1
X.104	-1	-1
X.105	-1	-1
X.106
X.107
X.108	1	-1	1
X.109
X.110
X.111	.	.	2	1	1	-1	-1
X.112	.	-2	.	1	-1	-1	1
X.113	-1	1	-1
X.114	.	-2	.	-1	1	1	-1
X.115
X.116	.	.	2	-1	-1	1	1
X.117	-1	1	-1
X.118
X.119	1	-1	1
X.120
X.121
X.122
X.123	-1	1	1
X.124	1	-1	-1
X.125	-1	1	1
X.126	1	-1	-1
X.127	-1	-1	-1	1	-1	1	1
X.128	-2	-2	-2	2	2	2
X.129	D	D	D	-1	D	-1	1
X.130	D	D	D	-1	D	-1	1
X.131	.	1	-3
X.132	.	-3	1
X.133	.	1	1
X.134	.	1	1
X.135
X.136
X.137
X.138
X.139
X.140	1	1	-1
X.141	1	-1	-1
X.142	-1	-1	-1
X.143	-1	1	1
X.144	-1	-1
X.145	2
X.146
X.147	.	.	2
X.148
X.149	-2
X.150	.	-2
X.151	.	.	.	-1	1	1	-1
X.152	.	.	.	-1	-1	1	1
X.153	.	.	.	1	-1	-1	1
X.154	.	.	.	1	1	-1	-1
X.155
X.156
X.157	-1	1	-1	-1	1	1	-1	1	-1	.	.
X.158	-1	-1	1	-1	-1	1	-1	-1	1	.	.
X.159	1	-1	1	-1	-1	1	-1	1	-1	.	.
X.160	-1	1	-1	-1	1	1	-1	-1	1	.	.
X.161	-1	-1	1
X.162	1	1	-1
X.163	1	-1	-1
X.164	-1	-1	1
X.165	-1	1	1
X.166	1	1	-1
X.167	1	-1	1	1	-1	-1	1
X.168	-1	1	-1	1	1	-1	1
X.169	D	D	D	-1	D	1	1
X.170	D	D	D	-1	D	1	1
X.171	-1	-1
X.172	-1	1
X.173	1	-1
X.174	1	1
X.175	-1	1
X.176	1	-1
X.177	.	.	2
X.178
X.179
X.180
X.181
X.182
X.183
X.184
X.185	2	.	-2	-2	.	.
X.186
X.187
X.188
X.189

where $A = -12\zeta(3)_3 - 6$, $B = 4\zeta(3)_3 + 2$, $C = 2\zeta(3)_3 + 2$, $D = 2\zeta(3)_3 + 1$, $E = -2\zeta(8)_8^3 - 2\zeta(8)_8$, $F = -24\zeta(3)_3 - 12$, $G = -8\zeta(3)_3 - 4$, $H = -16\zeta(3)_3 - 8$, $I = -48\zeta(3)_3 - 24$, $J = 4\zeta(3)_3 + 1$, $K = -36\zeta(3)_3 - 18$.

Character table of $D(2F_{i_{22}})$ (continued)

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	67	71	73	79	83	89	97	101	103	107	113	127	131	137	143	149	157	163	167	173	179	181	187	193	197	211	223	227	229	233	239	241	247	251	257	263	269	271	277	281	283	287	293	299	307	311	313	317	323	329	331	337	343	349	353	359	367	371	373	379	383	389	397	401	403	407	413	419	421	427	431	433	437	443	449	457	463	469	471	477	481	483	487	493	499	503	507	511	513	517	523	529	531	537	543	549	553	559	563	569	571	577	581	583	587	593	599	603	607	611	613	617	623	629	631	637	643	649	653	659	663	669	671	677	681	683	687	693	699	703	707	711	713	717	723	729	731	737	743	749	753	759	763	769	771	777	781	783	787	793	799	803	807	811	813	817	823	829	831	837	843	849	853	859	863	869	871	877	881	883	887	893	899	903	907	911	913	917	923	929	931	937	943	949	953	959	963	969	971	977	981	983	987	993	999	1003	1007	1011	1013	1017	1023	1029	1031	1037	1043	1049	1053	1059	1063	1069	1071	1077	1081	1083	1087	1093	1099	1103	1107	1111	1113	1117	1123	1129	1131	1137	1143	1149	1153	1159	1163	1169	1171	1177	1181	1183	1187	1193	1199	1203	1207	1211	1213	1217	1223	1229	1231	1237	1243	1249	1253	1259	1263	1269	1271	1277	1281	1283	1287	1293	1299	1303	1307	1311	1313	1317	1323	1329	1331	1337	1343	1349	1353	1359	1363	1369	1371	1377	1381	1383	1387	1393	1399	1403	1407	1411	1413	1417	1423	1429	1431	1437	1443	1449	1453	1459	1463	1469	1471	1477	1481	1483	1487	1493	1499	1503	1507	1511	1513	1517	1523	1529	1531	1537	1543	1549	1553	1559	1563	1569	1571	1577	1581	1583	1587	1593	1599	1603	1607	1611	1613	1617	1623	1629	1631	1637	1643	1649	1653	1659	1663	1669	1671	1677	1681	1683	1687	1693	1699	1703	1707	1711	1713	1717	1723	1729	1731	1737	1743	1749	1753	1759	1763	1769	1771	1777	1781	1783	1787	1793	1799	1803	1807	1811	1813	1817	1823	1829	1831	1837	1843	1849	1853	1859	1863	1869	1871	1877	1881	1883	1887	1893	1899	1903	1907	1911	1913	1917	1923	1929	1931	1937	1943	1949	1953	1959	1963	1969	1971	1977	1981	1983	1987	1993	1999	2003	2007	2011	2013	2017	2023	2029	2031	2037	2043	2049	2053	2059	2063	2069	2071	2077	2081	2083	2087	2093	2099	2103	2107	2111	2113	2117	2123	2129	2131	2137	2143	2149	2153	2159	2163	2169	2171	2177	2181	2183	2187	2193	2199	2203	2207	2211	2213	2217	2223	2229	2231	2237	2243	2249	2253	2259	2263	2269	2271	2277	2281	2283	2287	2293	2299	2303	2307	2311	2313	2317	2323	2329	2331	2337	2343	2349	2353	2359	2363	2369	2371	2377	2381	2383	2387	2393	2399	2403	2407	2411	2413	2417	2423	2429	2431	2437	2443	2449	2453	2459	2463	2469	2471	2477	2481	2483	2487	2493	2499	2503	2507	2511	2513	2517	2523	2529	2531	2537	2543	2549	2553	2559	2563	2569	2571	2577	2581	2583	2587	2593	2599	2603	2607	2611	2613	2617	2623	2629	2631	2637	2643	2649	2653	2659	2663	2669	2671	2677	2681	2683	2687	2693	2699	2703	2707	2711	2713	2717	2723	2729	2731	2737	2743	2749	2753	2759	2763	2769	2771	2777	2781	2783	2787	2793	2799	2803	2807	2811	2813	2817	2823	2829	2831	2837	2843	2849	2853	2859	2863	2869	2871	2877	2881	2883	2887	2893	2899	2903	2907	2911	2913	2917	2923	2929	2931	2937	2943	2949	2953	2959	2963	2969	2971	2977	2981	2983	2987	2993	2999	3003	3007	3011	3013	3017	3023	3029	3031	3037	3043	3049	3053	3059	3063	3069	3071	3077	3081	3083	3087	3093	3099	3103	3107	3111	3113	3117	3123	3129	3131	3137	3143	3149	3153	3159	3163	3169	3171	3177	3181	3183	3187	3193	3199	3203	3207	3211	3213	3217	3223	3229	3231	3237	3243	3249	3253	3259	3263	3269	3271	3277	3281	3283	3287	3293	3299	3303	3307	3311	3313	3317	3323	3329	3331	3337	3343	3349	3353	3359	3363	3369	3371	3377	3381	3383	3387	3393	3399	3403	3407	3411	3413	3417	3423	3429	3431	3437	3443	3449	3453	3459	3463	3469	3471	3477	3481	3483	3487	3493	3499	3503	3507	3511	3513	3517	3523	3529	3531	3537	3543	3549	3553	3559	3563	3569	3571	3577	3581	3583	3587	3593	3599	3603	3607	3611	3613	3617	3623	3629	3631	3637	3643	3649	3653	3659	3663	3669	3671	3677	3681	3683	3687	3693	3699	3703	3707	3711	3713	3717	3723	3729	3731	3737	3743	3749	3753	3759	3763	3769	3771	3777	3781	3783	3787	3793	3799	3803	3807	3811	3813	3817	3823	3829	3831	3837	3843	3849	3853	3859	3863	3869	3871	3877	3881	3883	3887	3893	3899	3903	3907	3911	3913	3917	3923	3929	3931	3937	3943	3949	3953	3959	3963	3969	3971	3977	3981	3983	3987	3993	3999	4003	4007	4011	4013	4017	4023	4029	4031	4037	4043	4049	4053	4059	4063	4069	4071	4077	4081	4083	4087	4093	4099	4103	4107	4111	4113	4117	4123	4129	4131	4137	4143	4149	4153	4159	4163	4169	4171	4177	4181	4183	4187	4193	4199	4203	4207	4211	4213	4217	4223	4229	4231	4237	4243	4249	4253	4259	4263	4269	4271	4277	4281	4283	4287	4293	4299	4303	4307	4311	4313	4317	4323	4329	4331	4337	4343	4349	4353	4359	4363	4369	4371	4377	4381	4383	4387	4393	4399	4403	4407	4411	4413	4417	4423	4429	4431	4437	4443	4449	4453	4459	4463	4469	4471	4477	4481	4483	4487	4493	4499	4503	4507	4511	4513	4517	4523	4529	4531	4537	4543	4549	4553	4559	4563	4569	4571	4577	4581	4583	4587	4593	4599	4603	4607	4611	4613	4617	4623	4629	4631	4637	4643	4649	4653	4659	4663	4669	4671	4677	4681	4683	4687	4693	4699	4703	4707	4711	4713	4717	4723	4729	4731	4737	4743	4749	4753	4759	4763	4769	4771	4777	4781	4783	4787	4793	4799	4803	4807	4811	4813	4817	4823	4829	4831	4837	4843	4849	4853	4859	4863	4869	4871	4877	4881	4883	4887	4893	4899	4903	4907	4911	4913	4917	4923	4929	4931	4937	4943	4949	4953	4959	4963	4969	4971	4977	4981	4983	4987	4993	4999	5003	5007	5011	5013	5017	5023	5029	5031	5037	5043	5049	5053	5059	5063	5069	5071	5077	5081	5083	5087	5093	5099	5103	5107	5111	5113	5117	5123	5129	5131	5137	5143	5149	5153	5159	5163	5169	5171	5177	5181	5183	5187	5193	5199	5203	5207	5211	5213	5217	5223	5229	5231	5237	5243	5249	5253	5259	5263	5269	5271	5277	5281	5283	5287	5293	5299	5303	5307	5311	5313	5317	5323	5329	5331	5337	5343	5349	5353	5359	5363	5369	5371	5377	5381	5383	5387	5393	5399	5403	5407	5411	5413	5417	5423	5429	5431	5437	5443	5449	5453	5459	5463	5469	5471	5477	5481	5483	5487	5493	5499	5503	5507	5511	5513	5517	5523	5529	5531	5537	5543	5549	5553	5559	5563	5569	5571	5577	5581	5583	5587	5593	5599	5603	5607	5611	5613	5617	5623	5629	5631	5637	5643	5649	5653	5659	5663	5669	5671	5677	5681	5683	5687	5693	5699	5703	5707	5711	5713	5717	5723	5729	5731	5737	5743	5749	5753	5759	5763	5769	5771	5777	5781	5783	5787	5793	5799	5803	5807	5811	5813	5817	5823	5829	5831	5837	5843	5849	5853	5859	5863	5869	5871	5877	5881	5883	5887	5893	5899	5903	5907	5911	5913	5917	5923	5929	5931	5937	5943	5949	5953	5959	5963	5969	5971	5977	5981	5983	5987	5993	5999	6003	6007	6011	6013	6017	6023	6029	6031	6037	6043	6049	6053	6059	6063	6069	6071	6077	6081	6083	6087	6093	6099	6103	6107	6111	6113	6117	6123	6129	6131	6137	6143	6149	6153	6159	6163	6169	6171	6177	6181	6183	6187	6193	6199	6203	6207	6211	6213	6217	6223	6229	6231	6237	6243	6249	6253	6259	6263	6269	6271	6277	6281	6283	6287	6293	6299	6303	6307	6311	6313	6317	632
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Character table of $D(2F_{i_{22}})$ (continued)

	2	9	9	9	9	9	8	8	8	8	8	8	8	8	7	7	7	7	7	7	3	6	6	6	6	6	6	6										
	3	1	1	1	1	1	1	1	1									
	5	1	1	1	1	1	1	1	1									
2P	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	5a	6a	6b	6c	6d	6e	6f	6g										
3P	215	214	211	213	216	210	216	220	219	220	220	223	216	219	223	227	228	227	220	228	5a	3a	3a	3a	3a	3a	3a	3a	3a									
5P	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	5a	215	216	22	217	23	21	218										
X.1	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	1a	6a	6b	6c	6d	6e	6f	6g										
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1						
X.3	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1					
X.4	2	.	2	.	.	2	.	2	2	.	2	.	.	2	-1	1	1	1	1	1	1	1	1	1	1	1	1	1				
X.5	-2	.	-2	.	.	-2	.	-2	-2	.	-2	.	.	-2	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
X.6	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
X.7	-1	1	1	1	1	-1	1	-1	-3	-3	1	1	-1	1	1	-1	-1	-1	-1	1	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2				
X.8	1	1	-1	1	1	-1	-1	3	3	1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
X.9	-2	.	-2	.	-2	.	-2	.	-2	.	-2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
X.10	2	.	2	.	-2	.	-4	-4	-2	.	2	.	-2	.	2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
X.11	-2	.	-2	.	2	.	-2	-2	-2	2	-2	.	2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.12	-2	.	-2	.	-2	.	2	2	2	-2	-2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.13	-2	.	-2	.	-2	.	2	2	2	-2	-2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.14	.	-2	.	-2	.	-2	.	-2	-2	-2	-2	.	-2	.	-2	.	-2	.	-2	.	2	2	-2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	
X.15	-1	-1	1	1	1	-1	1	-3	-3	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
X.16	1	-1	-1	-1	-1	1	1	3	3	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.17	-3	-1	-3	-1	-1	-3	-1	1	1	3	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.18	3	-1	3	-1	-1	3	-1	-1	-1	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.19	-1	1	1	1	1	1	1	1	1	3	-3	-1	1	-3	-1	1	1	1	1	1	-3	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.20	1	-1	-1	-1	1	-1	1	-1	-1	3	3	-1	1	3	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.21	1	3	1	3	3	-3	-1	-3	-1	-1	3	-1	3	-3	-1	1	1	1	1	1	3	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.22	-1	3	1	3	3	3	1	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.23	3	3	3	3	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.24	-3	3	-3	3	3	-3	3	3	3	1	1	-1	1	-1	3	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.25	-2	.	-2	.	2	.	2	.	2	.	2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.26	2	.	2	.	-2	.	2	.	2	.	2	.	-2	.	2	.	2	.	2	.	-2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.27	2	.	2	.	-2	.	2	.	2	.	2	.	-2	.	2	.	2	.	2	.	-2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.28	-2	.	-2	.	-2	.	2	.	2	.	2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.29	-2	.	-2	.	-2	.	2	.	2	.	2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.30	-2	.	-2	.	-2	.	2	.	2	.	2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.31	-2	-2	-4	2	.	-4	.	.	.	-2	4	2	.	-4	2	2	
X.32	-2	-2	4	2	.	4	.	.	.	-2	-4	2	.	-4	2	2	
X.33	-2	-2	-2	2	.	2	.	.	.	-2	2	2	.	-2	2	2	
X.34	-2	.	-2	.	-2	.	2	.	2	.	2	.	-2	.	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.35	4	2	-2	-2	.	-2	.	2	2	2	-2	-2	.	-2	-2	.	-2	.	-2	.	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
X.36	-4	2	2	-2	.	2	.	-2	-2	2	2	-2	.	2	-2	.	-2	.	-2	.	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2		
X.37	.	.	4	-4	2	-2	2	-2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2		
X.38	.	.	2	-2	.	-2	.	2	-2	2	-2	2	.	2	-2	.	-2	.	-2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.39	.	.	-2	2	.	2	.	-2	-2	-2	-2	-2	.	2	2	.	2	.	2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.40	.	.	-2	2	.	2	.	-2	-2	-2	-2	-2	.	2	2	.	2	.	2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.41	.	.	2	-2	.	2	.	-2	-2	-2	-2	-2	.	2	2	.	2	.	2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.42	.	.	2	-2	.	2	.	-2	-2	-2	-2	-2	.	2	2	.	2	.	2	.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.43	.	.	-2	2	.	-2	.	2	-2	2	-2	2	.	-2	-2	.	-2	.</																				

Character table of $D(2F_{122})$ (continued)

	2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	13	13	13	13	14	14
	1	1	1	1	1	1	1	1	1	1	1	.	.	1	1	.	1	1	1	1	.	.
	3	1	1	1	1	1	1	1	1	1	1	.	.	1	1	.	1	1	1	1	.	.
	5	1	1	1	1	1	1	1	1	1	1	.	.	1	1	.	1	1	1	1	.	.
	1a	21	22	23	24	25	26	27	28	29	210	211	212	213	214	215	216	217	218	219	220	
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P	1a	21	22	23	24	25	26	27	28	29	210	211	212	213	214	215	216	217	218	219	220	
5P	1a	21	22	23	24	25	26	27	28	29	210	211	212	213	214	215	216	217	218	219	220	
X.94	120	120	120	120	-120	-120	48	48	48	48	24	24	-48	-48	-24	12	12	12	12	-8	-8	
X.95	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8	-8	-8	
X.96	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8	-8	-8	
X.97	120	120	120	120	-120	-120	-48	-48	-48	-48	24	24	48	48	-24	12	12	12	12	-8	-8	
X.98	120	120	120	120	120	120	-24	-24	-24	-24	-8	-8	-24	-24	-8	8	8	
X.99	120	120	120	120	-120	-120	48	48	48	48	24	24	-48	-48	-24	12	12	12	12	-8	-8	
X.100	120	120	120	120	120	120	24	24	24	24	-8	-8	24	24	-8	8	8	
X.101	120	120	120	120	120	120	-24	-24	-24	-24	-8	-8	-24	-24	-8	8	8	
X.102	128	-128	-128	128	.	.	64	-64	64	-64	-32	32	32	-32	.	.	.	
X.103	160	-160	160	-160	160	-160	-16	-16	16	16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.104	160	-160	160	-160	160	-160	16	16	-16	-16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.105	160	-160	160	160	.	.	80	-80	80	-80	-40	40	40	-40	.	.	.	
X.106	160	-160	160	-160	160	-160	16	16	-16	-16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.107	160	-160	160	-160	160	-160	48	48	-48	-48	.	.	48	-48	.	8	8	-8	-8	.	.	
X.108	160	-160	160	-160	160	-160	-16	-16	16	16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.109	160	-160	160	-160	160	-160	-16	-16	16	16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.110	160	-160	160	-160	160	-160	16	16	-16	-16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.111	160	160	-160	-160	.	.	64	-64	-64	64	-32	32	.	.	-16	16	-16	16	-32	32	.	
X.112	160	-160	160	-160	160	-160	16	16	-16	-16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.113	160	160	-160	-160	-32	32	.	.	16	-16	16	-16	-32	32	.	
X.114	160	160	160	160	-160	-160	32	32	.	.	-32	-16	-16	-16	-32	32	.	
X.115	160	-160	160	160	.	.	-48	48	-48	48	-8	8	8	-8	.	.	.	
X.116	160	-160	160	-160	160	-160	-16	-16	16	16	.	.	-16	16	.	-8	-8	8	8	.	.	
X.117	160	-160	160	-160	160	-160	-48	48	48	48	.	.	48	-48	.	8	8	-8	-8	.	.	
X.118	160	160	-160	-160	.	.	-64	64	64	-64	-32	32	.	.	-16	16	-16	16	-32	32	.	
X.119	160	-160	160	160	.	.	-48	48	-48	48	-8	8	8	-8	.	.	.	
X.120	160	-160	160	160	.	.	-48	48	-48	48	-8	8	8	-8	.	.	.	
X.121	160	160	-160	-160	-32	32	.	.	16	-16	16	-16	-32	32	.	
X.122	160	160	-160	-160	-32	32	.	.	16	-16	16	-16	-32	32	.	
X.123	192	192	-192	-192	64	-64	
X.124	192	192	-192	-192	64	-64	
X.125	240	240	240	240	240	240	48	48	48	48	-16	-16	48	48	-16	16	16	
X.126	240	240	240	240	240	240	48	48	48	48	-16	-16	48	48	-16	-16	-16	
X.127	240	240	240	-240	.	.	96	-96	-96	96	-48	48	.	.	-24	24	-24	24	16	-16	.	
X.128	240	240	-240	-240	.	.	-96	96	96	-96	-48	48	.	.	-24	24	-24	24	16	-16	.	
X.129	240	-240	240	-240	-240	240	-72	-72	72	72	.	.	72	-72	.	12	12	-12	-12	.	.	
X.130	240	-240	240	-240	240	-240	72	72	-72	-72	.	.	72	-72	.	12	12	-12	-12	.	.	
X.131	240	-240	240	-240	240	-240	72	72	-72	-72	.	.	72	-72	.	12	12	-12	-12	.	.	
X.132	240	240	240	240	-240	-240	48	48	.	.	-48	-24	-24	-24	-24	-16	-16	
X.133	240	240	240	-240	.	.	-96	96	96	-96	-48	48	.	.	-24	24	-24	24	16	-16	.	
X.134	240	240	240	-240	.	.	96	-96	-96	96	-48	48	.	.	-24	24	-24	24	16	-16	.	
X.135	240	-240	240	-240	-240	240	-72	-72	72	72	.	.	72	-72	.	12	12	-12	-12	.	.	
X.136	240	240	240	240	240	240	-48	-48	-48	-48	-16	-16	-48	-48	-16	16	16	
X.137	240	240	240	240	-240	-240	48	48	.	.	-48	-24	-24	-24	-24	-16	-16	
X.138	240	240	240	240	240	240	-48	-48	-48	-48	-16	-16	-48	-48	-16	-16	-16	
X.139	240	240	240	240	240	240	-48	-48	-48	-48	-16	-16	-48	-48	-16	-16	-16	
X.140	320	-320	320	320	.	.	32	-32	32	-32	16	-16	-16	16	.	.	.	
X.141	320	-320	320	-320	320	-320	-32	-32	32	32	.	.	-32	32	.	-16	-16	16	16	.	.	
X.142	320	-320	320	-320	320	-320	32	32	-32	-32	.	.	-32	32	.	-16	-16	16	16	.	.	
X.143	320	-320	320	320	.	.	32	-32	32	-32	16	-16	-16	16	.	.	.	
X.144	320	-320	320	-320	320	-320	32	32	-32	-32	.	.	-32	32	.	-16	-16	16	16	.	.	
X.145	320	-320	320	-320	320	-320	-32	-32	32	32	.	.	-32	32	.	-16	-16	16	16	.	.	
X.146	384	384	384	384	-384	-384	-128	-128	.	.	128	
X.147	384	384	-384	-384	128	-128	
X.148	384	384	384	-384	128	-128	
X.149	480	-480	-480	480	.	.	-144	144	-144	144	-24	24	24	-24	.	.	.	
X.150	480	-480	-480	-480	-96	96	.	.	48	-48	48	-48	32	-32	.	
X.151	640	-640	-640	640	.	.	64	-64	64	-64	32	-32	-32	32	.	.	.	

Character table of $D(2\text{Fi}_{22})$ (continued)

	2	9	9	9	9	9	9	9	9	9	8	8	8	8	8	8	8	8	7	7	7	7	7	3	6	6		
	3																							1	1	1		
	5																							1	1	1		
	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	5a	6a	6b	
2P	214	216	213	216	215	214	211	213	216	210	216	220	219	220	220	223	216	219	223	227	228	227	220	228	5a	3a	3a	
3P	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	5a	215	216	
5P	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	1a	6a	6b	
X.94		-4		-2	4		6		-2	-6	2	2	-2		-2		2	2										
X.95			-4					-4																				
X.96			-4					-4																				
X.97		-4		2	4		-6		2	6	-2	-2	2		2		-2	-2										
X.98							-4			-4			-4		-4							4						
X.99		4		2	-4		-2		2	2	-2	2	-2		-2		2	2										
X.100	-4					-4	-4			-4									2	2	2		2					
X.101						-4				-4			-4		4			4				2	2	2	-4			
X.102																									-2	-2	2	
X.103																									1	1	1	
X.104		-4			4																				1	1	1	
X.105			-4	4				4	-4																2	-2	-2	
X.106																									1	1	1	
X.107				4					4		-4						4								2	2	2	
X.108		4			4																				1	1	1	
X.109		-4			-4																				1	1	1	
X.110																									1	1	1	
X.111																									2	-2	-2	
X.112		4			-4																				1	1	1	
X.113																									4	-4	-4	
X.114																									2	2	2	
X.115			-4	-4				4	4																4	-4	-4	
X.116																									1	1	1	
X.117				-4					-4		-4						4								2	2	2	
X.118																									2	-2	-2	
X.119			-4	-4				4	4																-2	2	2	
X.120			-4	-4				4	4																-2	2	2	
X.121																										-2	2	2
X.122							8																		2			
X.123	-8																								2			
X.124	8						-8																					
X.125	8						8																					
X.126			8					8																				
X.127	-4			4		4			-4						-4			4										
X.128	-4			-4		4			4						-4			4										
X.129		2		2	-2				2		2					-2			-2	2	2				-2			
X.130		2		2	2				-2		2					-2			-2	-2	2	2			2			
X.131		-2		-2	-2				-2		2					-2			2	2	-2	-2			-2			
X.132							-4			4		4	-4		4			-4										
X.133	4			4		-4			-4						4			-4										
X.134	4			-4		-4			4						4			-4										
X.135		-2		2	2				2		2					-2			2	-2	-2		2					
X.136														8														
X.137						4			-4		-4	4		-4				4										
X.138																												
X.139																												
X.140			-8					8																	-2	2	2	
X.141		4			4																				-1	-1	-1	
X.142		-4			4																				-1	-1	-1	
X.143			8					-8																	-2	2	2	
X.144		4			-4																				-1	-1	-1	
X.145		-4			-4																				-1	-1	-1	
X.146																									-1			
X.147																									-1			
X.148																									-1			
X.149			4	4			-4	-4																				
X.150																												
X.151																									2	-2	-2	

Character table of $D(2\text{Fi}_{22})$ (continued)

	2	6	6	6	6	6	5	5	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
	3								1	1	1	1	1	1	1	1	1	1	1	1	1		
	5								1	1	1	1	1	1	1	1	1	1	1	1	1		
	8l	8m	8n	8o	8p	8q	8r	8s	10a	10b	10c	10d	10e	10f	10g	12a	12b	12c	12d	12e	12f	16a	16b
2P	4 ₁₉	4 ₂₀	4 ₁₉	4 ₆	4 ₁₈	4 ₂₀	4 ₂₉	4 ₃₀	5a	5a	5a	5a	5a	5a	5a	6a	6b	6c	6a	6c	6b	8a	8a
3P	8l	8m	8n	8o	8j	8q	8r	8s	10a	10c	10b	10d	10e	10g	10f	4 ₁₃	4 ₁₄	4 ₁	4 ₁₅	4 ₂	4 ₁₆	16a	16b
5P	8l	8m	8n	8o	8p	8q	8r	8s	2 ₂	2 ₄	2 ₄	2 ₃	2 ₁	2 ₅	2 ₅	12a	12b	12c	12d	12e	12f	16b	16a
X.94
X.95	.	-2	.	.	.	-2
X.96	.	-2	.	.	.	-2
X.97
X.98	2	-2
X.99
X.100	-2
X.101	-2	2
X.102	2	.	.	.	-2	2
X.103	1	1	-1	-1	1	-1	.	.
X.104	-1	1	-1	1	1	-1	.	.
X.105	1	-1	1	-1	-1	1	.	.
X.106
X.107
X.108	-1	-1	1	1	-1	1	.	.
X.109	1	1	-1	-1	1	-1	.	.
X.110	-1	1	-1	1	1	-1	.	.
X.111
X.112	1	-1	1	-1	-1	1	.	.
X.113
X.114
X.115
X.116	-1	-1	1	1	-1	1	.	.
X.117
X.118
X.119
X.120
X.121
X.122
X.123	-2	.	.	.	-2	2
X.124	-2	.	.	.	-2	2
X.125
X.126
X.127	2	.	-2
X.128	-2	.	2
X.129
X.130	.	2	.	.	.	-2
X.131	.	-2	.	.	.	2
X.132
X.133	2	.	-2
X.134	-2	.	2
X.135
X.136
X.137
X.138	$\frac{D}{D}$
X.139	$\frac{D}{D}$
X.140	$\frac{D}{D}$
X.141	1	1	-1	-1	1	-1	.	.
X.142	1	-1	1	-1	-1	1	.	.
X.143
X.144	-1	1	-1	1	1	-1	.	.
X.145	-1	-1	1	1	-1	1	.	.
X.146	-1	1	1	-1	-1	1	1
X.147	1	B	-B	1	-1	-B	B
X.148	1	-B	B	1	-1	B	-B
X.149
X.150
X.151

where $A = -4\zeta(4)_4, B = -2\zeta(5)_5^3 - 2\zeta(5)_5^2 - 1, C = -4\zeta(3)_3 - 2, D = 2\zeta(8)_8^3 + 2\zeta(8)_8$.

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